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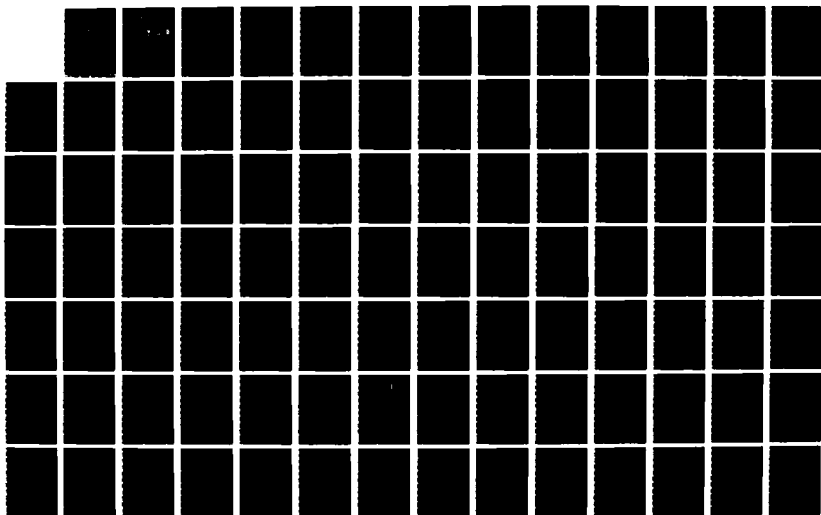
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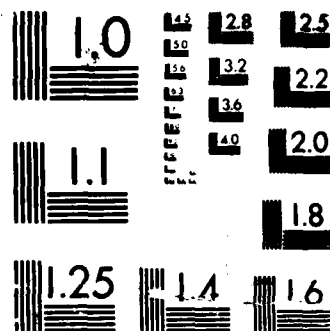
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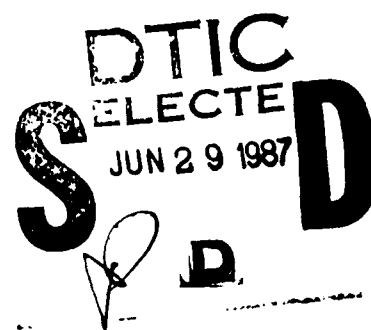
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THESIS

FURTHER DEVELOPMENT OF A
ONE-DIMENSIONAL UNSTEADY EULER CODE
FOR WAVE ROTOR APPLICATIONS

by

David T. Johnston

March 1987

Thesis Advisor:

Ray P. Shreeve

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Monterey, California

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Further Development of a
One-Dimensional Unsteady Euler Code
for Wave Rotor Applications

by

David T. Johnston
Lieutenant, United States Navy
B.S., Cornell University, 1979

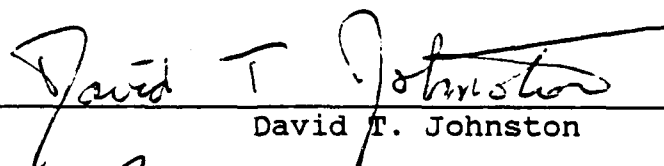
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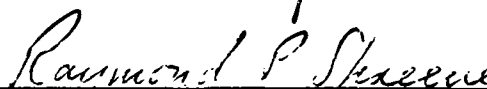
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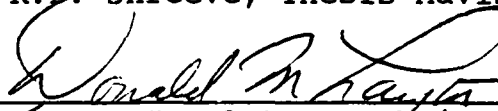
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ABSTRACT

The EULER1 Fortran program for computing one-dimensional unsteady flow based on the QAZ1D method of Verhoff was extended to provide tracking and correcting of discontinuities, flow to right or left and open and closed end boundary conditions. The program was run on four shock tube test cases to verify accuracy and range of capability of the revised E1DV2 code. Further extensions and test verifications are recommended so that the code is suitable for wave rotor applications.



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TABLE OF SYMBOLS

| <u>TEXT</u> | <u>E1DV2</u> | <u>DEFINITION</u> |
|-------------|--------------|--|
| A | A | Speed of Sound |
| | *A | Denotes the value of a variable at the node to the left of a discontinuity, * can be any variable name |
| A_i/A_j | AR | The ratio of sound speed across a shock. A/B (shock moving right), B/A (shock moving left) |
| | *B | Denotes the value of a variable at the node to the right of a discontinuity |
| | BDRY | 3 denotes left boundary, 2 the right boundary |
| | *BD | Variable at phantom node |
| | CNTACT | 3 digit variable denoting contact surface location, direction of travel, and if it crosses a node |
| | COUNT | Counter for graphics routines |
| | CSDIR | Contact surface direction, 2 to the right, 3 to the left |
| | CSRMN | Riemann variable change across a contact surface |
| | D2 | Density ratio at first node inside boundary |
| | DARRAY | Array of density for plotting |
| | DELAH | Change in A from I to I+1 |
| | DELAL | Change in A from I-1 to I |
| | DELQQH | Change in QQ from I to I+1 |
| | DELQQL | Change in QQ from I-1 to I |
| | DELRRH | Change in RR from I to I+1 |

| | | |
|-------------------|---------|---|
| | DELRRRL | Change in RR from I-1 to I |
| | DELSH | Change in S from I to I+1 |
| | DELSL | Change in S from I-1 to I |
| δt | DELT | Time step |
| δt_{ex} | DELTEX | The excess time in a time step when the shock is exactly at the solid wall |
| ∂t_{wl} | DELTWL | The time for the shock to reach the wall |
| | DELQH | Change in Q from I to I+1 |
| | DELQL | Change in Q from I-1 to I |
| Δs | DELX | Interpolation distance (LMD+DELT) |
| ρ | DENS | Density |
| | DLCD | Density to the left of the contact discontinuity in the exact solution |
| | DLSH | Density to the left of the shock in the exact solution |
| | DLTA** | Prefix which indicates the spatial change in ** for one time step |
| | DQ | The jump in velocity across the shock divided by the sound speed at B (right) or A (left) |
| | DR | The ratio of the density across a shock, B/A (right), B/A (left) |
| | DRI | Initial density ratio across the diaphragm |
| | EE | Desired precision for characteristic calculations |
| | E(K) | Actual error in characteristic slope calculation |
| | EREIMN | The jump in QQ across the shock calculated analytically as a function of w |
| γ | G | Gamma (ratio of specific heats) |

| | |
|----------|---|
| GRAPHS | For graphical output, 0 = none (tabular), 1 = plots all variables, 2 = compares density with exact solution |
| G1 | $1/(G-1)$ |
| G2 | $2/(G-1)$ |
| H | $1/(N-1)$ |
| HALT | Terminates program if 1, set by condi- tions not coded |
| INTEG(K) | Result of integrating Z(K) |
| **INT | Value of ** interpolated between nodes on the current time level |
| I2 | Number of the node to the right of a discontinuity |
| JSTOP | Number of time levels to be calculated |
| LBDDR | Left boundary density ratio |
| LBDDRI | Left boundary density ratio at time zero |
| LB DPR | Left boundary pressure ratio |
| LB DPRI | Left boundary pressure ratio at time zero |
| LB DPRS | Value of 0 denotes constant pressure at left boundary, 1 denotes adjustable pres- sure at the left boundary |
| LB DTR | Left boundary temperature ratio |
| LB DTRI | Left boundary temperature ratio at time zero |
| LBNDRY | Denotes left boundary condition, open or closed |
| LMD(K) | The characteristic trajectories in the space-time plane ($q+A$, $q-A$, q) |
| LNODE | Array of left most node to be corrected in CORRCT |
| LWPRES | Denotes which side of diaphragm has low pressure |

| | | |
|---------------------------------|----------|--|
| | LXX | Node defining the left interval |
| | MREIMN | The measured jump in QQ across the shock, from A to B |
| | N | Number of Spacial Nodes (odd number) |
| | ND | Double precision value of N |
| | NEW**(I) | Stored values of ** for the next time level |
| | PARRAY | Array of pressures for plotting |
| P_i/P_j | PR | The ratio of the pressure across a shock, A/B (right), B/A (left) |
| P | PRESS | Pressure |
| | PRI | Initial pressure ratio across the diaphragm |
| | PLTCNT | Counter for graphics routines |
| $\frac{\partial *}{\partial s}$ | *PRIM(K) | Suffix which indicates the spatial derivative of * at the current time level |
| | P2 | Pressure ratio at first node inside boundary |
| q | Q | Absolute fluid velocity |
| | QARRAY | Array of velocities for plotting |
| | QLBD | Initial velocity at left boundary |
| | QLI | Initial velocity left of the diaphragm |
| | QRBD | Initial velocity at right boundary |
| | QRI | Initial velocity right of the diaphragm |
| Q | QQ | $Q+A*S$ (extended Riemann variable) |
| | RBDDR | Right boundary density ratio |
| | RBDDRI | Initial right boundary density ratio |
| | RB DPR | Right boundary pressure ratio |
| | RBDPRI | Initial right boundary pressure ratio |

| | | |
|---|--------|---|
| | RBDPRS | Value of 0 denotes a constant pressure at right boundary, while 1 is for adjustable pressure |
| | RBDTR | Right boundary temperature ratio |
| | RBDTRI | Initial right boundary temperature ratio |
| | RBNDRY | Denotes right boundary open or closed |
| | RNODE | Array for right most node to be corrected in CORRCT |
| R | RR | $Q-A*S$ (extended Riemann variable) |
| | RXX | Node defining the right interval |
| S | S | Entropy |
| | SAP | Entropy to the left of the shock for flows right or entropy to the right of the C.S. for flows left |
| | SARRAY | Array of entropy for plotting |
| | SAVG | Average entropy |
| | SBP | Entropy to the right of the C.S. for flows right or entropy to the left of the shock for flows left |
| | SHKDIR | Shock direction of travel, 3 to the left, 2 to the right |
| | SHOCK | 3 digit variable denoting shock location, direction of travel, and if it crosses a node |
| | SIGMA | Spatial location of discontinuities Sigma(L,J) where L indicates the type of discontinuity and J indicates the time level: 1--current level, 2--level being calculated |
| | SK | Integer that denotes relative location of shock near boundaries |
| | SKIP | Variable which indicates how many time steps between calls to output routines |
| | **STEP | The change in time of ** at a node used to step up to the next time level |

| | | |
|----------------|--------|---|
| t | T | Time since initial conditions |
| T | TEMP | Temperature |
| | TRI | Initial temp, ratio across the diaphragm |
| | TS | Time for shock to travel one interval |
| | T2 | Temperature ratio at first node inside boundary |
| u _A | UA | Velocity relative to the shock, left side |
| u _B | UB | Velocity relative to the shock, right side |
| | VHEAD | Velocity of the head of the expansion wave for the exact solution |
| | VCDE | Velocity of the contact discontinuity for the exact solution |
| V _S | VS | The shock speed (positive right, negative left) |
| | VSE | Velocity of the shock for the exact solution |
| | VTAIL | Velocity of the tail of the expansion wave for the exact solution |
| w | W | Mach no. relative to a standard shock |
| $\frac{w}{w}$ | | Vector of the principle variables, Q,R,S |
| X | X | Location in spacial plane (I-1)*H |
| | XARRAY | Array of spatial positions for plotting |
| | XEXACT | Array of six X values for the exact solution |
| | XINIT | Initial position of discontinuity for exact solution plotting |
| | X2 | Location of node to right of discont. along the spacial axis |
| | Y | (N+1)/2 |
| | YEXACT | Array of six density values for the exact solution |

| | | |
|----------------|--------|---|
| \bar{Z} | $Z(K)$ | Vector of the right hand sides of the governing equations |
| Δ | | Small spatial change |
| δ | | Small change with respect to time |
| θ, ϕ | | Flow angles with respect to reference coordinate planes |

ACKNOWLEDGEMENT

Thanks to my best friend, Gayle LeVeque, for her support and encouragement, to Atul Mathur, for his invaluable assistance, and to Ray Shreeve, for making this a challenging and rewarding learning experience.

The work is dedicated in memory of my grandparents, Ray Litton Johnston and Anna Elizabeth Weech.

I. INTRODUCTION

The investigation of wave rotors¹ and their possible application in gas turbine engines at the Naval Postgraduate School's (NPS) Turbopropulsion Laboratory prompted the development of a one-dimensional unsteady flow computer code. The QAZ1D method developed by Verhoff [Ref. 1] was chosen over other methods [Ref. 2] for reasons which included the following:

1. The method is based on the use of characteristics. Such methods can model wave propagation accurately.
2. The use of a natural streamline coordinate system eases the difficult task of computing with two and three dimensional grids.
3. The equations are written in a form which allows a straightforward extension to viscous flows.

Salacka [Ref. 2] verified the development of the equations and implemented a one-dimensional Euler solution of the

¹The wave rotor consists of simple tube-like passages along and around the periphery of a drum which rotates between end walls containing inlet and outlet (partial admission) gas ports. Compression and expansion processes are arranged to occur in a cyclically periodic unsteady process within the rotor such that a high pressure driver gas is used to compress a low pressure driven gas. The compressed driven gas is led from the outlet port to the combustor and brought back into the rotor as the driver. The expanded driver gas exits an outlet port. Thus the rotor functions as both turbine and compressor with direct gas-gas energy exchange through unsteady wave processes. The technology of such devices requires the design of appropriate cycles using one-dimensional calculations, and analysis of the multi-dimensional unsteady flows such as occur during port opening and closing.

shock tube problem in the EULER1 code. The EULER1 code was limited to high pressure set on the left, with flow and shock velocities to the right. The shock wave was tracked and corrected for in the flow, but the contact discontinuity was not, and the expansion wave was not tracked. The code also did not treat end boundary conditions.

The present work extended the EULER1 code of Salacka to become the "Euler 1D Version 2" or E1DV2 code which:

- 1) can handle both right and left traveling flow and discontinuities
- 2) implements tracking and correction of the contact discontinuity
- 3) implements tracking of the expansion wave
- 4) models closed wall and open end boundary conditions
- 5) models shock and contact surfaces within a grid interval.

Section II and Appendix A describe the analysis underlying the revisions. The Fortran code, E1DV2, is detailed in Section III and Appendix C and graphical results of four test cases to demonstrate the new features of the code are given in Section IV. A discussion of these results and limitations of the present code follow in Section V. Conclusions and recommendations are made in Section VI. Appendix B contains instructions to operate the code on the NPS computer and the FORTRAN listing is included as Appendix D.

II. ANALYTICAL AND COMPUTATIONAL APPROACH

A. QAZ1D DESCRIPTION

The QAZ1D analysis and numerical solution scheme is an explicit, non-conservative method that uses extended Riemann variables to model the multi-dimensional Euler equations in a natural streamline coordinate system. Table I lists a summary of the applicable equations which are derived in detail in [Ref. 2].

1. The Coordinate System

Figure 2.1 shows the natural streamline coordinate system (s,n,m) relative to a fixed rectangular cartesian system (x,y,z) . The right-hand orthogonal system moves in curvilinear translation along a streamline. The "s" direction is measured in the direction of the flow, tangent to the streamline. The "n" direction is perpendicular to the s direction in a plane perpendicular to the x-z plane. The "m" direction is normal to the plane containing the n and s directions. The angle ϕ is the angle between the s-n and x-y planes, and the angle θ is the angle between the velocity vector and the x-z plane in the s-n plane. In the one-dimensional problem treated here, the s direction is such that velocity to the right is positive, and to the left is negative. [Ref. 1:p. 2]

TABLE I
SUMMARY OF THE EULER EQUATIONS

Definitions

Speed of sound $A = \sqrt{\gamma P / \rho}$ (I.1)

Modified entropy change $ds = - \frac{dQ_R}{\gamma T}$ (I.2)

so that

$$\frac{\bar{S}}{R_G} = S = \frac{1}{\gamma(\gamma-1)} [2\gamma - \ln(P/\rho^\gamma)] \quad (I.3)$$

Extended Riemann variables

$$Q = q + AS \quad (I.4)$$

$$R = q - AS \quad (I.5)$$

Euler Equations

$$\begin{aligned} \frac{\partial Q}{\partial t} + (q+A) \frac{\partial Q}{\partial s} = & - \frac{\gamma-1}{2} A \left(S - \frac{2}{\gamma-1} \right) \left[\frac{\partial}{\partial s} \left(q - \frac{2}{\gamma-1} A \right) \right] \\ & - \frac{\gamma-1}{2} q A S \left[\frac{\partial \theta}{\partial n} + \cos \theta \frac{\partial \phi}{\partial m} \right] \end{aligned} \quad (I.6)$$

$$\begin{aligned} \frac{\partial R}{\partial t} + (q-A) \frac{\partial R}{\partial s} = & \frac{\gamma-1}{2} A \left(S - \frac{2}{\gamma-1} \right) \left[\frac{\partial}{\partial s} \left(q + \frac{2}{\gamma-1} A \right) \right] \\ & + \frac{\gamma-1}{2} q A S \left[\frac{\partial \theta}{\partial n} + \cos \theta \frac{\partial \phi}{\partial m} \right] \end{aligned} \quad (I.7)$$

$$\frac{\partial S}{\partial t} + q \frac{\partial S}{\partial s} = \phi \quad (I.8)$$

$$\frac{\partial \theta}{\partial t} + q \frac{\partial \theta}{\partial s} = - \frac{A^2}{\gamma q} \frac{\partial \ln P}{\partial n} \quad (I.9)$$

$$\frac{\partial \phi}{\partial t} + q \frac{\partial \phi}{\partial s} = - \frac{A^2}{\gamma q \cos \theta} \frac{\partial \ln P}{\partial m} \quad (I.10)$$

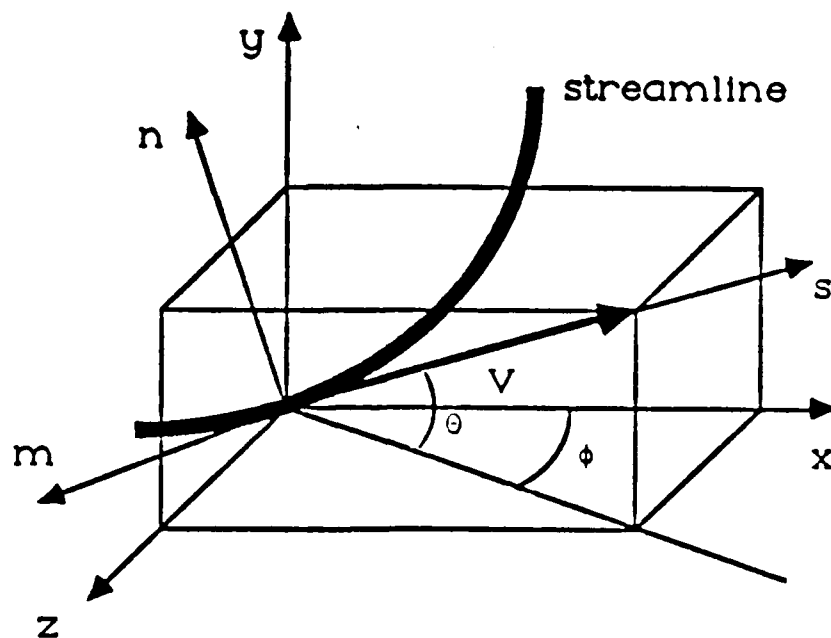


Figure 2.1 The Natural Coordinate System

2. Extended Riemann Variables

The "Riemann Variables" in most publications are defined as

$$R_1 = q - \left(\frac{2}{\gamma+1}\right) A$$

and

(2.1)

$$R_2 = q + \left(\frac{2}{\gamma+1}\right) A$$

where q is the velocity magnitude, A is the speed of sound, and γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume for a particular gas

[Ref. 3:p. 5]. Verhoff modified the definition to obtain "Extended Riemann Variables," defined as

$$Q = q + AS \quad (I.4)$$

$$R = q - AS \quad (I.5)$$

where S is the modified entropy given in Table I [Ref. 1:p. 2].

3. The Governing Equations

The Euler equations in Table I were developed in detail [Ref. 2:Appendix A] from the equations for conservation of mass, momentum, and energy for a control volume. The underlying assumptions comprise inviscid flow, negligible body forces, no heat transfer, and a perfect gas. These assumptions and the definition of modified entropy were used in transforming the equations into a form suitable for solution using the method of characteristics.

B. CHARACTERISTIC SOLUTION

Equations (I.6), (I.7), (I.8), (I.9) and (I.10) are a system of slightly coupled quasi-one-dimensional partial differential equations (PDE) with eigenvalues $q+A$, q , and $q-A$. These equations can be rewritten along the characteristics in the time-space domain to become ordinary differential equations (ODE) which are solved by quadrature and interpolation. The propagation of each characteristic

curve along its particular trajectory is expressed by the respective ODE. [Ref. 1:p. 1]

Each in the system of equations is in the form

$$\frac{\partial w}{\partial t} + \lambda \frac{\partial w}{\partial s} = z \quad (2.2)$$

If $w = w(s, t)$ it is shown by Salacka [Ref. 2:pp. 20-23] that

$$\lambda = \frac{ds}{dt} \quad (2.3)$$

and

$$\frac{dw}{dt} = z \quad (2.4)$$

where λ describes the characteristic curve along which w changes.

In this section the solution of the 3D Euler equations given in Table I is described. The computer code developed in the present work, however, was for the 1D case wherein equations (I.9) and (I.10) are not required, and only the three remaining equations (I.6), (I.7), and (I.8) are considered.

Equations (I.6) to (I.8) for one-dimensional flow may be expressed in matrix form by setting

$$\bar{w} = \begin{bmatrix} Q \\ R \\ S \end{bmatrix} \quad [\lambda] = \begin{bmatrix} q+A & 0 & 0 \\ 0 & q-A & 0 \\ 0 & 0 & q \end{bmatrix}$$

and

$$\bar{z} = \begin{bmatrix} -\frac{\gamma-1}{2}A(S - \frac{2}{\gamma-1})[q - \frac{2}{\gamma-1}A]_s \\ \frac{\gamma-1}{2}A(S - \frac{2}{\gamma-1})[q + \frac{2}{\gamma-1}A]_s \\ 0 \end{bmatrix}$$

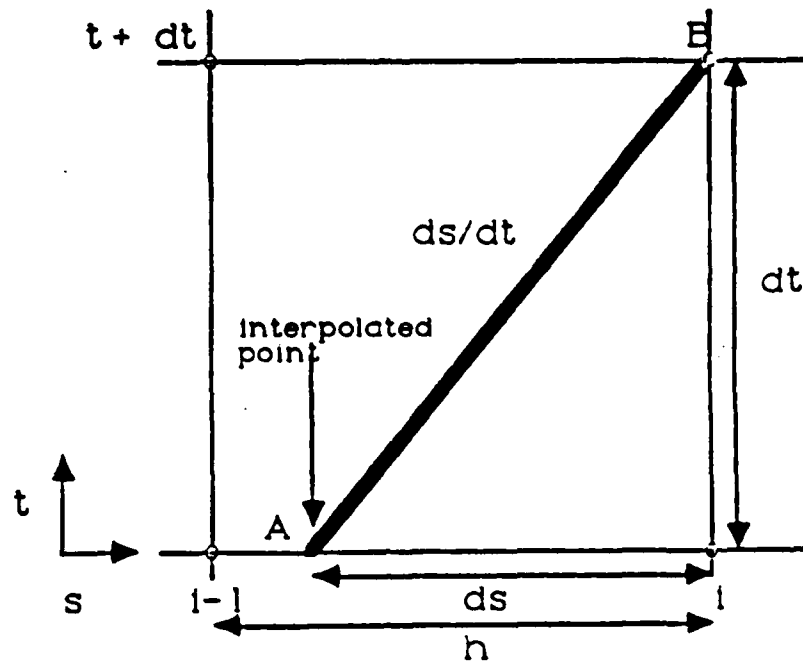
Then equation (2.2) becomes, for the equation set,

$$\frac{\partial \bar{w}}{\partial t} + [\lambda] \frac{\partial \bar{w}}{\partial s} = \bar{z}.$$

Figure 2.2 shows the characteristic curve in the space-time domain between two nodes within the computational mesh. Equation (2.2) has a solution

$$\delta \bar{w} = -\Delta \bar{w} + \int_t^{t+dt} \bar{z} dt \quad (2.5)$$

where $\delta \bar{w}$ is the change due to time at a fixed location, and $\Delta \bar{w}$ is the change due to displacement at a fixed location, along the characteristic curve. The value of $\Delta \bar{w}$ is calculated by interpolation through an iterative scheme



$$\delta \bar{w} = \bar{w}_B - \bar{w}_{s_i}$$

$$\Delta \bar{w} = \bar{w}_{s_i} - \bar{w}_A$$

Figure 2.2 Characteristic Curve Within Computational Mesh

using initial guesses for the characteristic slope, λ , from values at time level t . The line integral is approximated by

$$\begin{aligned} \int_t^{t+dt} \bar{z} dt &= \int_A^B \frac{\bar{z}}{\lambda} ds = \int_{s_i - \Delta s}^{s_i} \frac{\bar{z}}{\lambda} ds = \frac{\bar{z}}{\lambda} (s_i - (s_i - \Delta s)) \\ &= \bar{z} (\Delta s / \lambda) \\ &= \bar{z} (\delta t) \end{aligned} \quad (2.6)$$

The spatial derivatives of q and A in \bar{Z} are estimated from values at A and s_1 . Finally, $\bar{\partial w}$ is calculated, and each node is updated to the next time level.

The Courant-Friedrichs-Lewy (CFL) convergence condition requires the domain of dependence of the numerical approximation to include the domain of dependence of the differential equation [Ref. 4:pp. 336-337]. Figure 2.3 shows that

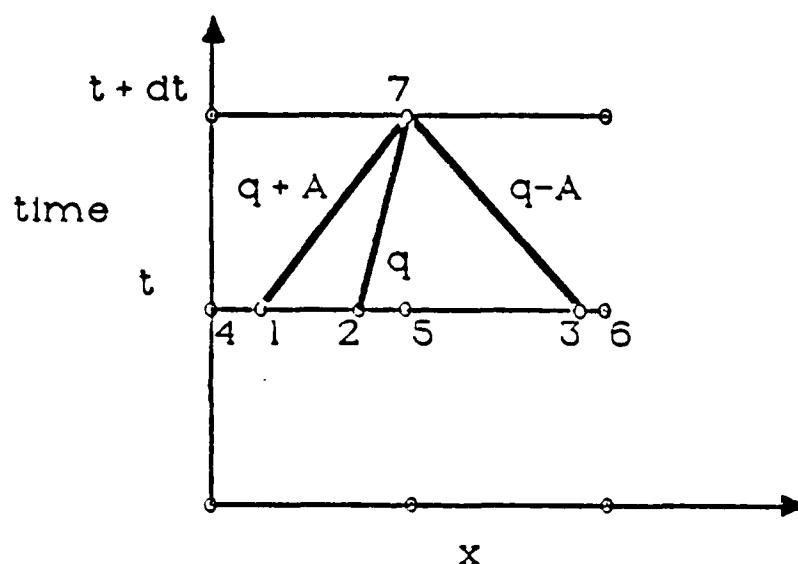


Figure 2.3 Computational Grid for the QAZ1D Method

the computational grid for the QAZ1D method satisfies this condition by having interpolated initial data points (points 1, 2, and 3) in between previous solution nodes (points 4, 5, 6). The $q+A$ characteristic is estimated from values at point 4, q characteristic from values at point 5, and $q-A$ characteristic from values at point 6. The use of a maximum

time step keeps the domain of dependence of the numerical scheme just outside that of the actual equations, aiding convergence. Unlike finite difference schemes that are conservative and require a numerical diffusion or viscosity to handle oscillations or smearing, this method is stable without numerical smoothing or artificial dissipation [Ref. 5:pp. 562-583]. However, the equations do not account for the discontinuities across shocks and contact surfaces properly. Thus after each time step a one-point correction technique is used to correct for shocks and contact surfaces.

C. DISCONTINUITIES

1. Expansion/Rarefaction Waves

The head and tail of the expansion wave travel along the characteristic as gradient discontinuities. Flow velocity, speed of sound, pressure, and density are continuous across a gradient discontinuity but the spatial derivatives are discontinuous. Entropy remains constant through the rarefaction wave. The head of an expansion wave moves into undisturbed flow with a speed $q+A$, while the tail of the expansion wave has a velocity $q-A$. No special treatment of these waves is required; the characteristics method accurately tracks and models their influence. Expansion waves are generated when two shocks collide, a diaphragm is burst, a contact surface and shock intersect, a shock leaves

an open boundary, or an open boundary has a pressure lower than the pressure inside the tube. [Ref. 3:pp. 13-15]

2. Shocks

Shocks are created when a diaphragm is burst, compression waves generated at an open boundary coalesce into a wavefront with sufficient strength, or when a shock and contact surface collide. Pressure, velocity, density, and entropy are discontinuous across a shock [Ref. 6:pp. 30-31]. The equations of motion (I.6)-(I.10) are not correct for calculating changes through shocks. A method to correct for a shock is to use the ratio of the jump in the extended Riemann variable across the shock to the speed of sound downstream of the shock to find the incoming Mach Number relative to the shock (w) [Ref. 1:p. 3]. Figure 2.4 shows the case of a shock headed to the right. The shock velocity

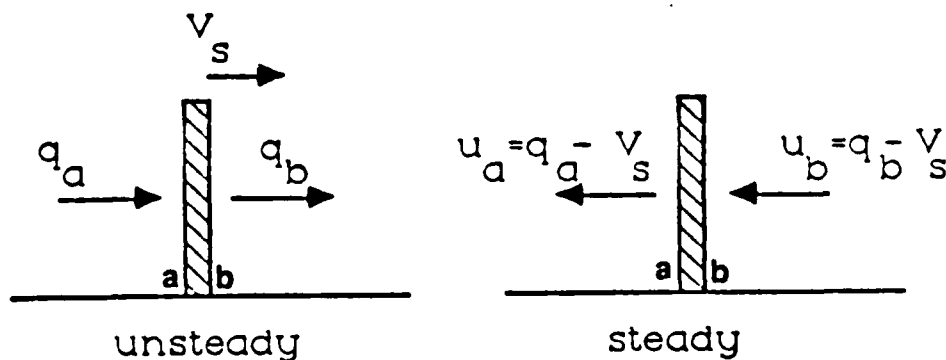


Figure 2.4 Shock Wave with High Pressure on Left

(V_s) is imposed on the flow to produce a stationary shock condition. Flow variables are then corrected using normal shock relations. Figure 2.4 applies to a condition where high pressure is on the left. The shock is moving to the right at V_s into flow with velocity q_B . Table II lists the equations developed by Salacka [Ref. 2] for relating the extended Riemann variable change to Mach Number. Velocities are non-dimensionalized by the speed of sound downstream of the shock direction, A_B , and pressures and densities by the respective values downstream.

Figure 2.5 depicts equation (II.2) over a range of Mach Numbers from 1.0 to 4.0. The curve is approximated by the quadratic equation

$$\frac{Q_A - Q_B}{A_B} = -2.7574 + 3.1573w - 0.2863w^2 \quad (2.7)$$

If the high pressure is to the right the shock moves as Fig. 2.6 illustrates, and the governing equations are as listed in Table III. Again, downstream conditions are used to non-dimensionalize velocity, pressure, density, and the Riemann variable change. The development of the equations in Table III is given in Appendix A. A plot of equation (III.2) over a range of w from 1.0 to 4.0 is shown in Fig. 2.7. The curve is approximated by the quadratic equation

$$\frac{R_B - R_A}{A_A} = 2.7574 - 3.15732w + 0.2863w^2 \quad (2.8)$$

TABLE II

SHOCK EQUATIONS FOR HIGH PRESSURE ON LEFT

Definitions

Relative Incoming
Mach Number: $w = -\frac{u_B}{\lambda_B} = -\frac{(q_B - V_s)}{\lambda_B}$ (II.1)

Equations

Q Riemann
Variable Change:

$$\frac{Q_A - Q_B}{\lambda_B} = \frac{2(w^2 - 1)}{(\gamma + 1)w} + \left(\frac{2}{\gamma - 1}\right) \left[\frac{\lambda_A}{\lambda_B} - 1\right] - \frac{\lambda_A}{\lambda_B} \left[\frac{1}{\gamma(\gamma - 1)}\right] \ln \left[\left(\frac{2\gamma}{\gamma + 1} w^2 - \frac{\gamma - 1}{\gamma + 1}\right) \left(\frac{(\gamma - 1)w^2 + 2}{(\gamma + 1)w^2}\right)^\gamma \right] \quad (\text{II.2})$$

Speed of Sound Ratio:

$$\frac{\lambda_A}{\lambda_B} = \frac{1}{(\gamma + 1)w} \left[2(\gamma - 1) \left[1 + \frac{\gamma - 1}{2} w^2 \right] \left[\frac{2\gamma}{\gamma - 1} w^2 - 1 \right] \right]^{1/2} \quad (\text{II.3})$$

Pressure Ratio:

$$\frac{P_A}{P_B} = \frac{2\gamma}{\gamma + 1} w^2 - \frac{\gamma - 1}{\gamma + 1} \quad (\text{II.4})$$

Density Ratio:

$$\frac{\rho_A}{\rho_B} = \frac{(\gamma + 1)w^2}{(\gamma - 1)w^2 + 2} \quad (\text{II.5})$$

TABLE II (CONTINUED)

Velocity Change:

$$\frac{q_A - q_B}{A_B} = \frac{2(w^2 - 1)}{(\gamma + 1)w} \quad (\text{II.6})$$

Entropy Change:

$$s_A - s_B = -\left(\frac{1}{\gamma(\gamma - 1)}\right) \ln \left[\frac{2\gamma w^2 - \gamma + 1}{\gamma + 1} \right] - \left(\frac{1}{\gamma - 1}\right) \ln \left[\frac{(\gamma - 1)w^2 + 2}{(\gamma + 1)w^2} \right] \quad (\text{II.7})$$

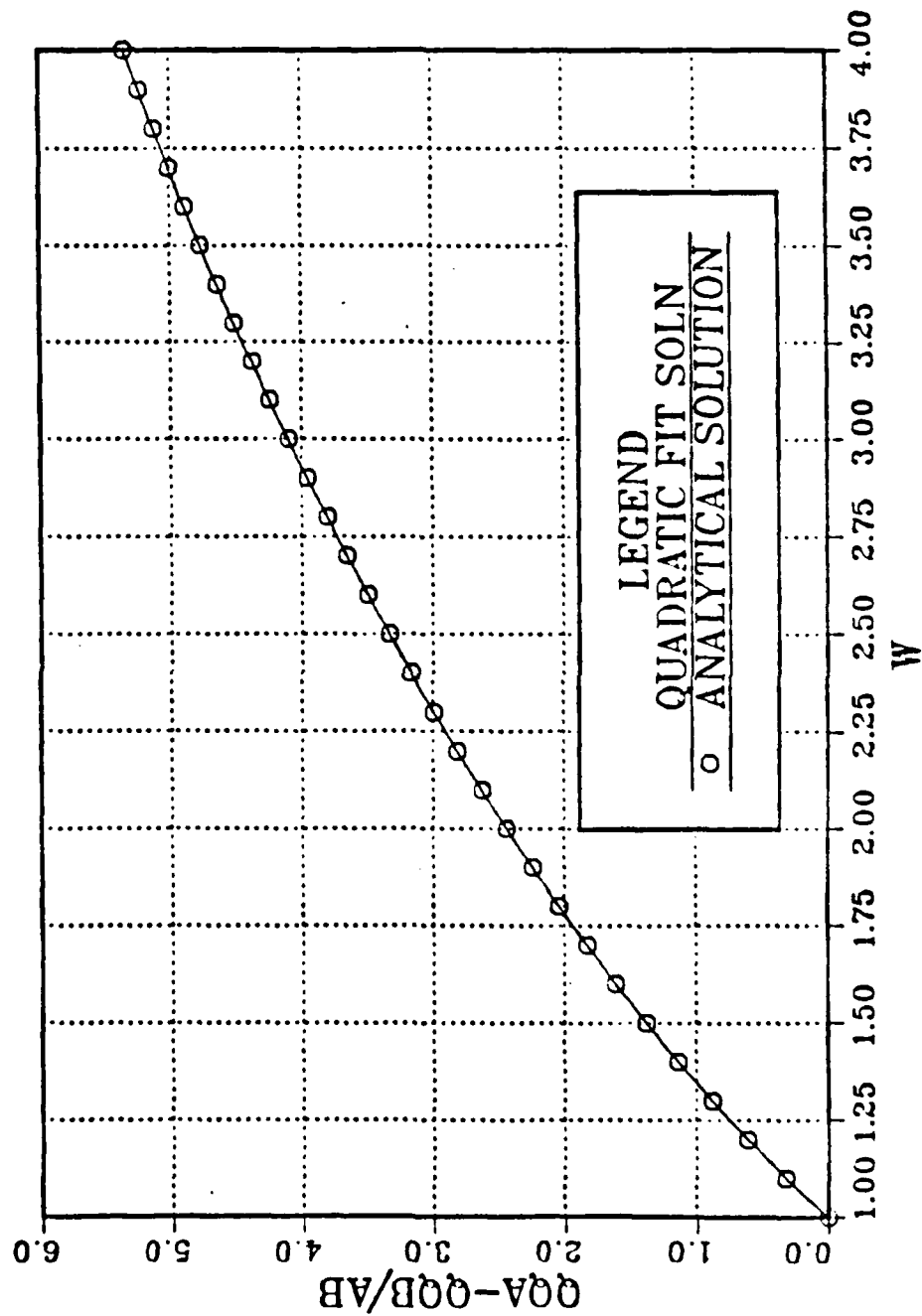


Figure 2.5 Q , Extended Riemann Variable Change with Mach Number

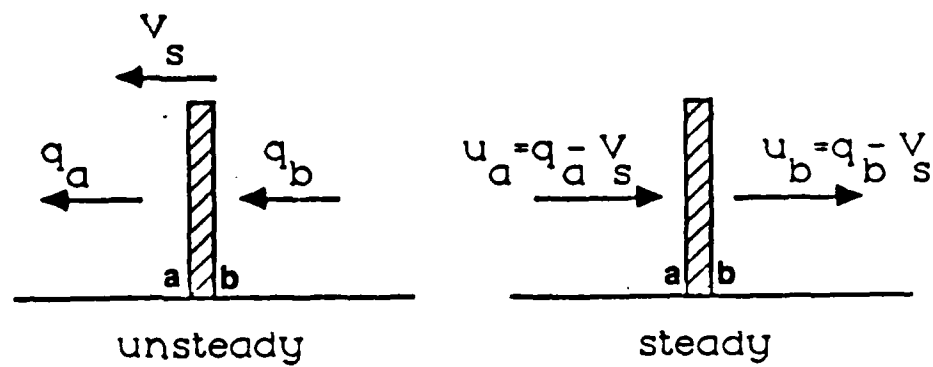


Figure 2.6 Shock Wave with High Pressure on Right

TABLE III

SHOCK EQUATIONS FOR HIGH PRESSURE ON RIGHT

Definitions

Relative Incoming
Mach Number:

$$w = \frac{u_A}{A_A} = \frac{q_A - V_s}{A_A} \quad (\text{III.1})$$

Equations

R Riemann
Variable Change:

$$\begin{aligned} \frac{R_B - R_A}{A_A} = & \frac{2(1-w^2)}{(\gamma+1)w} + \left(\frac{2}{\gamma-1}\right) \left[1 - \frac{A_B}{A_A}\right] + \frac{A_B}{A_A} \left[\frac{1}{\gamma(\gamma-1)}\right] \ln \left[\left(\frac{2\gamma}{\gamma+1}\right) w^2\right. \\ & \left. - \left(\frac{\gamma-1}{\gamma+1}\right) \left(\frac{(\gamma-1)w^2+2}{(\gamma+1)w^2}\right)^\gamma\right] \end{aligned} \quad (\text{III.2})$$

Speed of Sound Ratio:

$$\frac{A_B}{A_A} = \frac{1}{(\gamma+1)w} \left[2(\gamma-1) \left[1 + \frac{\gamma-1}{2} w^2\right] \left[\frac{2\gamma}{\gamma-1} w^2 - 1\right]\right]^{1/2} \quad (\text{III.3})$$

Pressure Ratio:

$$\frac{P_B}{P_A} = \frac{2\gamma}{\gamma+1} w^2 - \frac{\gamma-1}{\gamma+1} \quad (\text{III.4})$$

Density Ratio:

$$\frac{\rho_A}{\rho_B} = \frac{(\gamma+1)w^2}{(\gamma-1)w^2 + 2} \quad (\text{III.5})$$

TABLE III (CONTINUED)

Velocity Change:

$$\frac{q_B - q_A}{A_A} = \frac{2(1-w^2)}{(\gamma+1)w} \quad (\text{III.6})$$

Entropy Change:

$$s_B - s_A = -\left(\frac{1}{\gamma(\gamma-1)}\right) \ln\left[\frac{2\gamma w^2 - \gamma + 1}{\gamma + 1}\right] - \left(\frac{1}{\gamma-1}\right) \ln\left[\frac{(\gamma-1)w^2 + 2}{(\gamma+1)w^2}\right] \quad (\text{III.7})$$

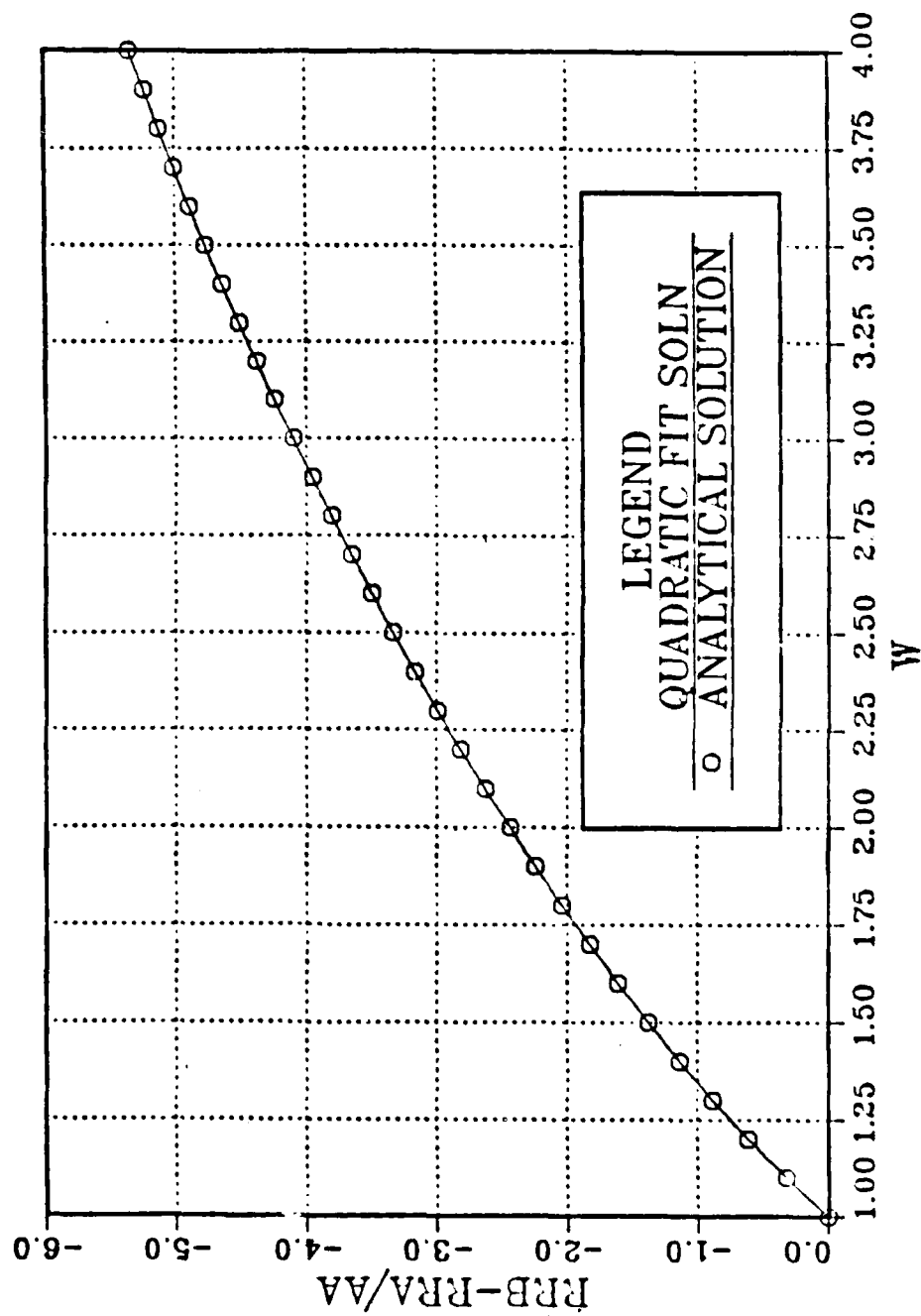


Figure 2.7 Extended Riemann Variable Change with Mach Number, High Pressure on Left

An examination of equations (II.2) and (III.2) reveals that

$$\frac{R_B - R_A}{A_A} = - \left[\frac{Q_A - Q_B}{A_B} \right]$$

This is expected since in flow to the right, the Q extended Riemann variable is associated with the q+A characteristic. Also, since velocity is defined as positive to the right, q is positive. But for flow to the left, velocity is defined as negative, thus q is negative. Moretti [Ref. 3] showed that the downstream running Riemann variable (q+A characteristic) has a lower magnitude of discontinuity across a normal shock than the upstream running Riemann variable (q-A characteristic), and is thus preferable. In flow to the left, the Riemann variable associated with downstream is the R Riemann variable, though the characteristic associated with the R Riemann variable is q-A, in flow to the left q is negative and thus

$$q-A = -(|q| + A) \quad (2.9)$$

Therefore, in fluid traveling left the R Riemann variable is used to find Mach Number, and in fluid traveling right the Q Riemann variable is used.

If the shock is between two nodes with no contact surface within the same interval, the method to calculate the Mach Number, w , is as follows:

- 1) The change in the appropriate extended Riemann variable across the interval is measured; R variable for flow left, and Q variable for flow right. Call this change, Δm .
- 2) Use Δm in equation (2.7) or equation (2.8) for the appropriate flow to calculate w .
- 3) With equation (II.2) or equation (II.3), for flow right or left respectively, use w to calculate the analytical Riemann variable change, Δe .
- 4) If this exact value of Δe is not equal to Δm then calculate a new guess, Δg from

$$\Delta g_{i+1} = \Delta g_i + (\Delta m - \Delta e) \quad (2.10)$$

and then repeat steps 2 to 4 until Δe equals Δm .

Thus the correct value of w is determined, and shock corrections from the normal shock relations are valid. V_s is determined from equation (II.1) or equation (III.1) depending on flow direction. Flow direction will be initially determined by the side with high pressure. High pressure on the left means flow will travel to the right. Likewise, high pressure on the right means flow will travel to the left.

3. Contact Surface

A contact surface is a boundary between regions of flow which are different in composition or which have undergone different thermodynamic histories. Thus a contact surface is formed when the diaphragm is burst in a shock

tube separating the gas initially in the driver from the gas initially in the driven tube [Ref. 6:pp. 23-29]. In a wave rotor, a contact surface would separate the combustor gases from the cooler inlet air. Contact surfaces are also caused by two shocks of opposite families colliding or a shock overtaking a shock of the same family [Ref. 3:pp. 15-18].

Figure 2.8 illustrates the changes in physical properties through the contact surface in the shock tube problem. The gas to the right has been compressed and heated by the shock wave, with that to the left cooled and

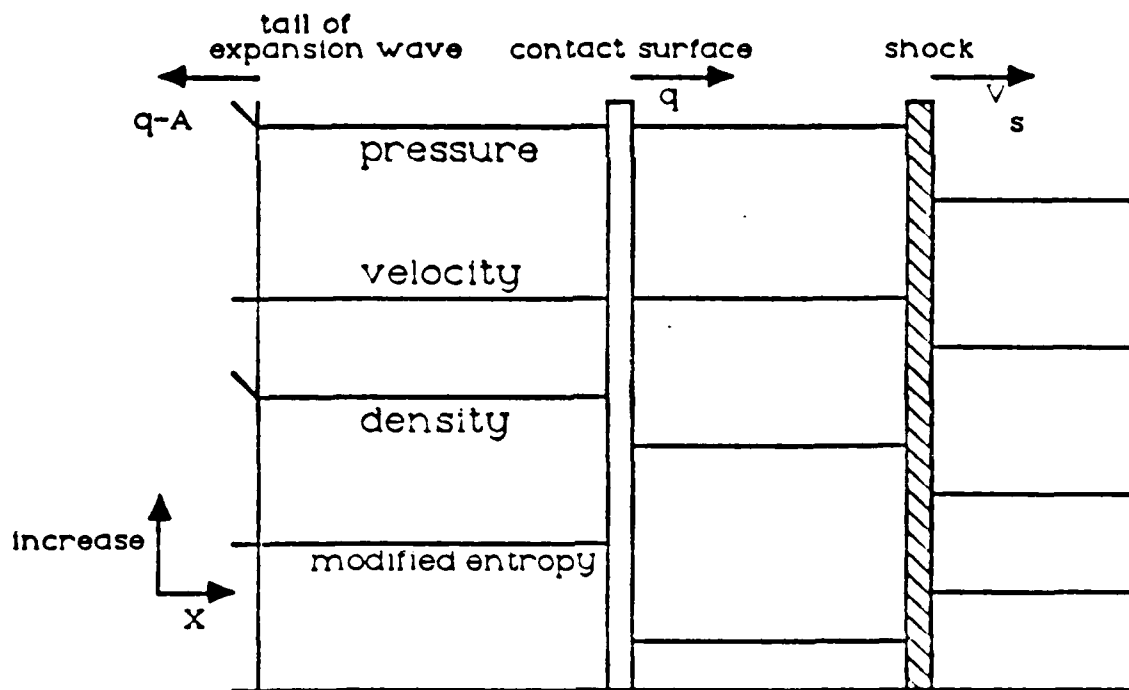


Figure 2.8 Behavior of Physical Properties Through a Contact Surface

expanded. The density, speed of sound, and modified entropy are discontinuous across the contact surface.

Since pressure is constant across the contact surface, using the perfect gas equation of state and the first law of thermodynamics it is shown in Appendix A for the shock tube problem that

$$A_A/A_B = \exp\left(\frac{(\gamma-1)}{2}(S_B-S_A)\right) \quad (2.11)$$

Since the flow behind the contact surface to the tail of the rarefaction wave is isentropic, the value of S_B will be known. The value of S_A is the value of entropy behind the shock. Subtracting equations (I.4) and (I.5) from each other and dividing by entropy S_B , A_B can be determined. Thus the unknown speed of sound A_A is obtained from equation (2.11). With q_B equal to q_A , and S_A the Riemann variables just behind the contact surface are calculated from equations (I.4) and (I.5). For a contact surface traveling to the left it is only necessary to interchange the subscripts. The velocity of the contact surface is easily determined since it moves at velocity q along the q characteristic line associated with S . [Ref. 3:p. 16]

4. Contact Surface/Shock Interaction

When the shock and contact surface are within the same interval the calculation of the shock incoming Mach Number, w , must be modified. Figure 2.9 shows a plot of the

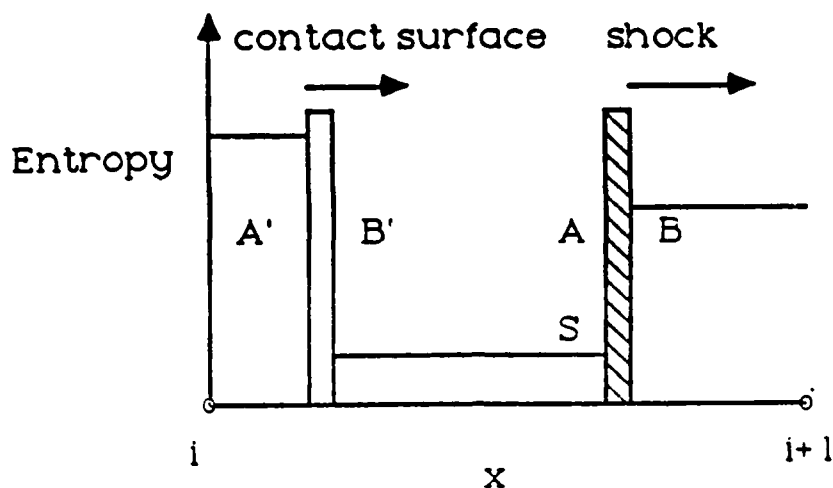


Figure 2.9 Modified Entropy Distribution for Shock and Contact Surface Traveling Right

modified entropy change across the interval. The use of the Riemann values at the nodes to estimate the Riemann variable change across the shock would be incorrect. The correct Riemann variable change would be, for flow to the right,

$$\Delta SK = \frac{Q_S - Q_B}{A_B} \quad (2.12)$$

The correct Mach Number, w , is determined using the technique of Moretti modified for QAZ1D [Ref. 3:pp. 23-24].

Referring to Fig. 2.9, the technique for flow to the right is as follows:

- 1) Calculate w as if no contact surface were in the interval. With values at node $i+1$ known, determine S_B . Then use w and S_B in equation (II.7) to solve for S_A . This is the initial estimate of S_S . Let $S_B' = S_S$.
- 2) The Q Riemann Variable change across a contact surface is shown in Appendix A to be given by

$$\Delta_{CS} = \frac{Q_{A'} - Q_{B'}}{A_B} = [e^{[(S_{B'} - S_{A'}) \frac{(\gamma-1)}{2}]} S_{A'} - S_{B'}] \frac{A_A}{A_B} \quad (2.13)$$

3) Define

$$\Delta m = \frac{Q_{A'} - Q_{B'}}{A_B} \quad (2.14)$$

then

$$\Delta m = \Delta SK + \Delta CS \quad (2.15)$$

can be solved to obtain ΔSK .

- 4) Using ΔSK as Δm in the shock iteration scheme given in Section II.C.2 calculate a new w .
- 5) With this new w and S_B use equation (II.7) to obtain a new S_A .
- 6) Compare S_A with S_S . If they are not equal to within an acceptable error, calculate a new estimate for S_S using

$$S_{S_{i+1}} = S_{S_i} + (S_A - S_S) \quad (2.16)$$

Iterate until convergence is achieved.

This will result in the proper w for a condition with shock and contact surface interacting.

For flow to the left, it is only necessary to interchange the subscripts A and B, and use

$$\frac{R_{B'} - R_{A'}}{A_A} = [\exp \frac{(\gamma-1)}{2} (S_{A'} - S_{B'}) S_{B'} - S_{A'}] \left(\frac{A_B}{A_A} \right) \quad (2.17)$$

for the extended Riemann Variable change. Figure 2.10 illustrates the left traveling condition.

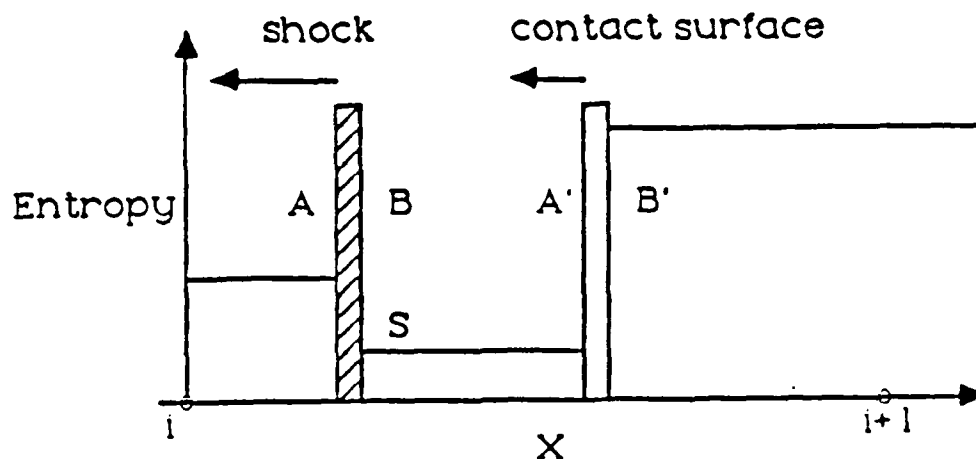


Figure 2.10 Modified Entropy Distribution for Shock and Contact Surface Traveling Left

D. BOUNDARY CONDITIONS

1. Open Boundary

At the open boundary, a reference pressure ratio, $P_{ref} = P_{\infty}/P_A$ is specified, where P_{∞} is the pressure to be held at the boundary, and P_A is the pressure just inside the tube. Only the case where $P_{\infty} \leq P_A$ is considered. This means that an outflow condition at the boundary will result when $P_{\infty} < P_A$, with an expansion wave traveling in from the boundary. Thus locally at the boundary, conditions are isentropic.

The computational grid for the left boundary is shown in Fig. 2.11. A phantom node, L, is used outside the

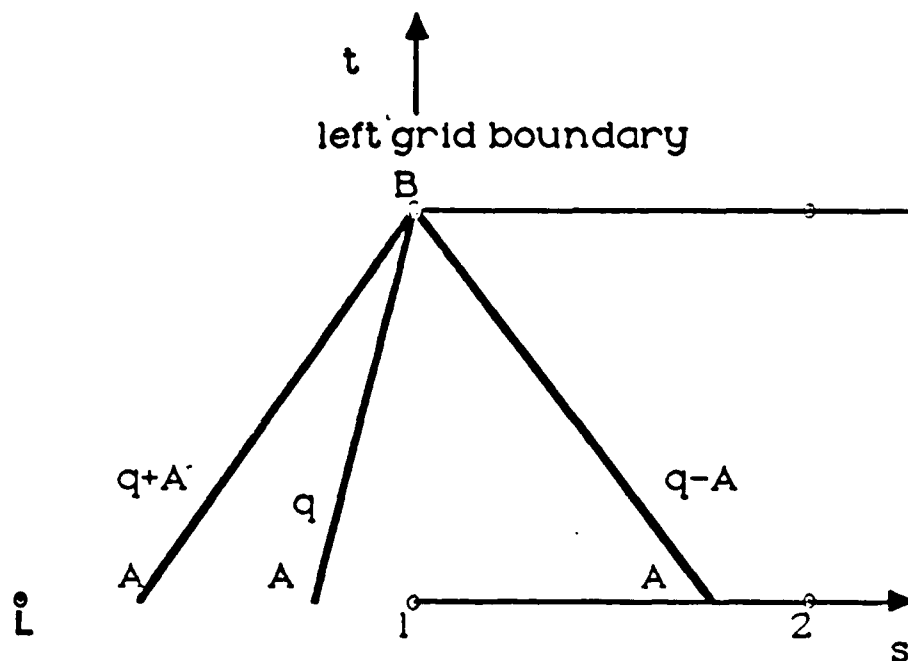


Figure 2.11 Left Boundary Computational Grid

computational mesh to enable simple enforcement of constant pressure and entropy at the mesh boundary node, 1.

The approach is to determine the required variables at node L, then transfer these values onto node 1, and vary q at the boundary to meet the required boundary conditions. First, using P_{ref} at node L and S at node 2 in equation (I.3) rewritten in the form

$$\rho_R = ((1/P_{ref}) (\exp^{(S - \frac{2}{\gamma-1}) (-\gamma(\gamma-1))})^{-1/\gamma}) \quad (2.18)$$

ρ_R , the density ratio at the boundary, is calculated. Using the perfect gas equation of state in equation (2.18) the temperature ratio, T_R is given by

$$T_R = (\rho_R^{(\gamma-1)}) (\exp^{(\gamma(1-\gamma)(S - \frac{2}{\gamma-1})}) \quad (2.19)$$

and can be calculated once ρ_R is known. With an initial velocity at the boundary, q_b , assumed the Riemann variables R and Q are determined for node L using

$$R = q_b - \sqrt{T_R} S \quad (2.20)$$

and

$$Q = q_b + \sqrt{T_R} S \quad (2.21)$$

Subtracting equation (2.20) from (2.21), and dividing by twice S yields A at node L , where $A = \sqrt{\gamma R_G T}$. The values of R , Q , A , q_b , and S are now known at node L . These values are assigned to node 1, and the QAZ1D algorithm is applied to the boundary node as if it were an interior node. The result is an estimate of the new values for R , Q , and S at node 1. These are used to obtain a new estimate of the pressure ratio at node 1, call it P_{RN} . P_{RN} is compared with P_{ref} , and the new value of S with the old value of S at node 1. A new q_b is calculated by solving equations (2.20)

and (2.21) simultaneously. The process is repeated until entropy and pressure remain constant at the boundary. For the right-hand boundary, Fig. 2.12 shows the computational

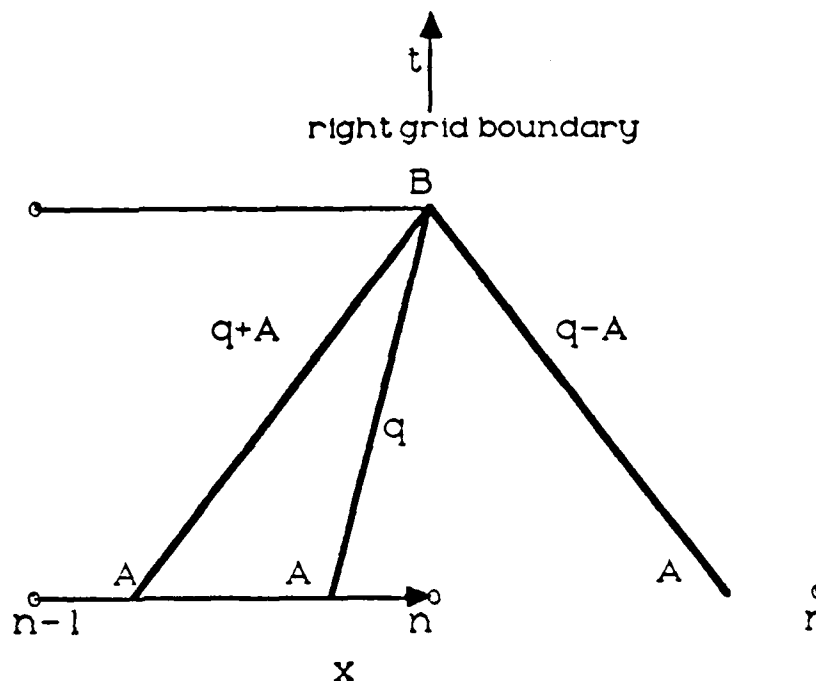


Figure 2.12 Right Boundary Computational Grid

grid with the phantom node, r , to the right of the mesh boundary node, n . The procedure for the right boundary is the same as for the left boundary.

When a shock travels across the open boundary, the boundary is first corrected using the method described in Section II.C.2. Subsequently, the pressure is returned to give the constant value specified for P_{ref} , using the above

procedure. The result will be an expansion wave traveling inward.

For supersonic flow, all of the variables are determined from values at the interior nodes [Ref. 1:p. 3].

2. Closed Boundary

The solid boundary is imposed by setting the velocity at the boundary, q , equal to zero. The same computational grid is used as that for an open boundary, but a different technique is used in assigning values to the phantom node. Referring to Fig. 2.11, the velocity at node 2, q_2 , is known. By setting

$$q_L = -q_2 \quad (2.22)$$

the wave impacting the boundary will be met by a wave of equal velocity but opposite in direction and will result in node 1 appearing as a solid boundary. Further, to facilitate the QAZ1D method, set

$$R_L = -Q_2 \quad (2.23)$$

$$Q_L = |R_2| \quad (2.24)$$

$$S_L = S_2 \quad (2.25)$$

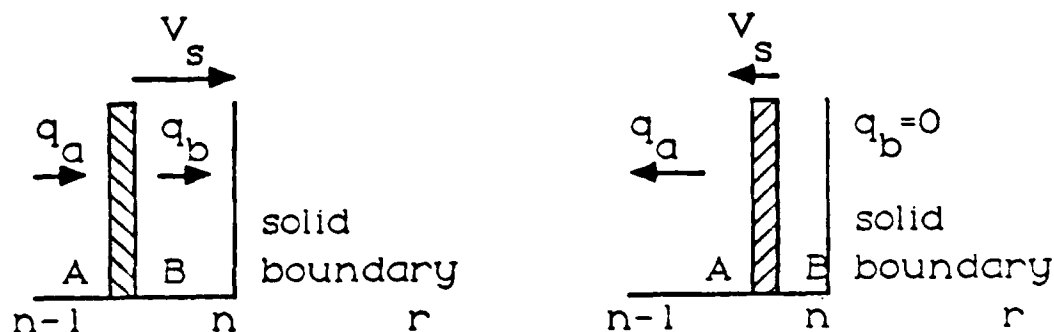
$$A_L = 2A \quad (2.26)$$

This will enable the computation of the new boundary node 1 values as an interior point with

$$q_1 = 0$$

The right boundary is handled in the same way.

Figure 2.13 illustrates the sequence of events when a shock reflects off a solid boundary on the right [Ref. 7:



A) Shock Approaches Solid Boundary

B) Shock Reflects Off Solid Boundary

Figure 2.13 Shock Reflecting at Solid Boundary

pp. 172-195]. Since the velocity behind the reflected shock, q_B , is zero, and the velocity in front of the shock, q_A , is known, the velocity gradient, Δq , over the shock is, in non-dimensionalized form, given by

$$\Delta q = (q_B - q_A)/A_A \quad (2.27)$$

Rearranging equation (III.6), and using $\gamma = 1.4$ it can be shown that

$$w = \sqrt{1 + 0.36(\Delta q)^2} - 0.6\Delta q \quad (2.28)$$

This is an exact analytical value for w based on stationary normal shock relations. The speed of sound ratio, A_B/A_A , is calculated from equation (III.3) and

$$A_B = A_A(A_B/A_A) \quad (2.29)$$

Entropy behind the shock, S_B , is determined from equation (III.7) since S_A is known. Then the extended Riemann variables, R and Q , behind the reflected shock are calculated from equations (I.4) and (I.5) respectively.

For a shock reflected from the left boundary, the subscripts A and B are interchanged and using equation (II.6) the Mach Number is given by

$$w = \sqrt{1 + 0.36(\Delta q)^2} + 0.6\Delta q \quad (2.30)$$

The equations from Table II are used instead of those from Table III (used in the case of reflection from the right boundary), since the shock is now traveling from left to right.

III. FORTRAN PROGRAM E1DV2

A. GENERAL DESCRIPTION

Program E1DV2 is a Fortran computer program which calculates 1-dimensional unsteady flow based on the QAZ1D formulation and method of solution of the Euler equations of motion. The program has provisions for tracking in time the location of shock waves, contact surfaces and head and tail of the expansion waves. Its intended application to wave-rotor design and analysis explains the present definition of node points and boundary conditions. In its current form, the code describes flow in a tube initiated by a diaphragm bursting. The location of the diaphragm, whether the flow is to the left or the right, and whether the ends of the tube are closed or open, can be varied.

The E1DV2 program is written in FORTRAN 77, and was developed using the IBM 3033 System 370 computer. The use of DISSPLA subroutines in plotting results requires compiling and executing using VS FORTRAN procedures. Appendix B describes the procedures for running the E1DV2 program on the Naval Postgraduate School IBM computer system. A listing of the code, which contains its own description in comment statements, is given in Appendix D.

The code incorporates:

- 1) first order accuracy
- 2) double precision numerics, except for single precision in graphical output
- 3) linear interpolation and extrapolation algorithms
- 4) quadratic polynomial approximations for curve fitting
- 5) non-dimensional variable input and output
- 6) structured subroutine format.

Table IV lists all the subroutines called from the main program and brief descriptions of their purposes. The extensive use of subroutines was intended to allow modifications and extensions of the code without major restructuring.

B. THE MAIN PROGRAM

The main program flowchart is illustrated in Appendix C, Fig. C.1. The conventions and a comprehensive list of variables are listed in the beginning of the program. The number of grid nodes is set by the user, and must be an odd number. Arrays are dimensioned to the number of nodes in the main program prior to compiling and executing.

Initial values for temperature, pressure, and density ratios at the diaphragm and boundaries are entered by editing. Also, the initial velocity distribution on each side of the diaphragm is set, and as well as the ratio of specific heats. The side with the high pressure is identified. Boundaries are set either closed or open. In the

TABLE IV
SUBROUTINES

| | |
|--------|--|
| TIME | Calculates minimum time step |
| TRAK | Tracks discontinuities, calculates shock mach number, w |
| SWEEP | Advances node values to next time step |
| COND1 | Calculates interpolated values of Riemann variables, $\partial q / \partial s$, and $\partial A / \partial s$ when no shock or contact surface within an interval |
| COND2 | Calculates interpolated values of Riemann variable, $\partial q / \partial s$, and $\partial A / \partial s$ when discontinuity within interval to right of node or flow is supersonic to the right |
| COND3 | Calculates interpolated values of Riemann variable, $\partial q / \partial s$, and $\partial A / \partial s$ when discontinuity within interval to left of node or flow is supersonic to the left |
| COND4 | Called when node is jumped by discontinuity to load node information into array for use in subroutine CORRECT |
| COND5 | Calculates interpolated values of Riemann variable, $\partial q / \partial s$, and $\partial A / \partial s$ when discontinuity on either side of node and flow is supersonic |
| COND6 | Would contain subroutine to advance node when contact surface is in front of shock after intersecting and traveling in same direction. Not currently in use |
| COND7 | Would contain subroutine to solve precisely when and where shock and contact surface would intersect within an interval. Not currently in use |
| COND7N | Would contain subroutine to solve precisely when and where shock and contact surface would intersect if either were jumping a node simultaneously. Not currently in use |

TABLE IV (CONTINUED)

| | |
|--------|--|
| COND7S | Would contain the subroutine to solve the Riemann problem of shock and contact surface intersecting. Not currently in use |
| COND8 | Would contain subroutine to advance node after shock and contact surface have crossed and are moving apart. Not currently in use |
| CORRCT | Advances nodes jumped by discontinuity to next time step |
| DELTAX | Determines relative location of discontinuity within an interval |
| SKJUMP | Calculates variable change across a normal stationary shock |
| CSJUMP | Calculates variable change across a contact surface |
| EXTRAP | Extrapolates entered values |
| INTERP | Interpolates entered values |
| DBURST | Calculates initial Riemann values at node where diaphragm is located |
| BONDRY | Calculates values to update left and right boundary |
| SRFLCT | Calculates new shock parameters when shock reflects off solid boundary |
| BBDRY | Alternate interpolating scheme near boundary |
| BORDER | Sets graphical borders |
| PLOT | Plots q , S , P , and ρ |
| EXACT | Plots exact ρ versus calculated ρ |
| LIST | Lists calculated values in files 8, 9, 10 |

open case, either constant pressure, or an adjustable pressure is specified. The latter case in effect extends the tube, and allows discontinuities to disappear out of the tube without creating expansion or compression waves. Exact values for velocities of expansion wave, shock, and contact surface, and density ratio are specified by the user in the input section.

Output is controlled by setting the variable GRAPHS to either 0, 1, or 2. A zero creates three files which contain data calculated at the time the call to subroutine LIST is made. A value of 1 causes plots of pressure, density, velocity, and entropy distributions to be created. When GRAPHS equals 2 a plot of the exact density distribution along with the calculated density distribution is made. Calls to output routines are determined by the parameter, SKIP, the number of time steps between calls, which is specified by the user.

Program termination can occur for several reasons. First, the program terminates when a maximum number of time steps, JSTOP, is reached. JSTOP is specified by the user. Second, the program terminates if, during execution, any of the following conditions are met:

- 1) The shock intersects with the contact surface
- 2) The pressure at any open boundary is higher than the pressure at the first node inside the boundary
- 3) The shock exits an open boundary.

In these cases a message is displayed on the terminal describing the reason for termination.

C. THE SUBROUTINE PROGRAMS

1. The "DBURST" Routine

Figure C.2 in Appendix C shows the flowchart for "DBURST". The assumption that a shock and contact surface are formed immediately causes oscillating numerical solutions that dampen within five to ten time steps. By mathematically "bursting" the diaphragm prior to time zero, a solution at the node where the diaphragm is located can be determined at time zero. The result is a more accurate representation of the wave structure and a dampening of a reduced transient solution within three time steps. "DBURST" calculates the Riemann variables that would exist behind a shock and contact surface that have moved an infinitesimal amount after the diaphragm burst, and assigns them to the node where the diaphragm was situated.

2. The "TIME" Routine

The "TIME" subroutine was not changed from that developed by Salacka [Ref. 2:p. 35]. The purpose is to compute the maximum allowable time step but ensuring that all characteristic trajectories over the time step are within one interval between nodes. The time step is determined from

$$\text{DELT} = \text{H}/\text{ABS}[Q+A] \quad (3.1)$$

at every node, and the minimum time step is used.

3. The "TRAK" Routine

This routine computes the new locations of the shock, contact surface, and head and tail of the expansion wave after the time step, DELT, calculated in "TIME." In Appendix C, Fig. C.3 shows the flowchart for the "TRAK" subroutine. The shock velocity, V_s , incoming Mach Number, w , and pressure, density, and sonic velocity ratios PR , DR , and AR respectively are determined. The equations for flow left and right are the same, except for DQ , the velocity gradient, which is different only by sign. Thus by proper bookkeeping the same equations are coded once and used in either direction. The initial direction of the shock and contact surface are set from code that interprets which side has the high pressure at time zero. The contact surface speed is determined after the shock speed is established. The situation at time zero is handled as in "DBURST" to ensure that the shock/contact surface interaction is properly modeled. Since there is a certain transient solution that decays after three time steps, tracking of the expansion wave is not initiated until then.

The head of the expansion wave travels at a velocity corresponding to $q+A$, and the tail with speed $q-A$. In the case of flow traveling to the left, this is reversed.

4. The "SWEEP" Routine

The "SWEEP" subroutine solves equation (2.5) and updates the values of variables at the nodes, including the boundary nodes. Appendix C, Fig. C.4 shows the "SWEEP" flowchart. Only those nodes which have been crossed by a discontinuity during the time step are updated by the "CORRECT" subroutine. Section II.B describes the theory coded into "SWEEP". The routine begins at the left boundary node and progresses to the right boundary node, one node at a time. Figure 3.1 illustrates the overall algorithm applying the QAZ1D method. At the boundary nodes, the

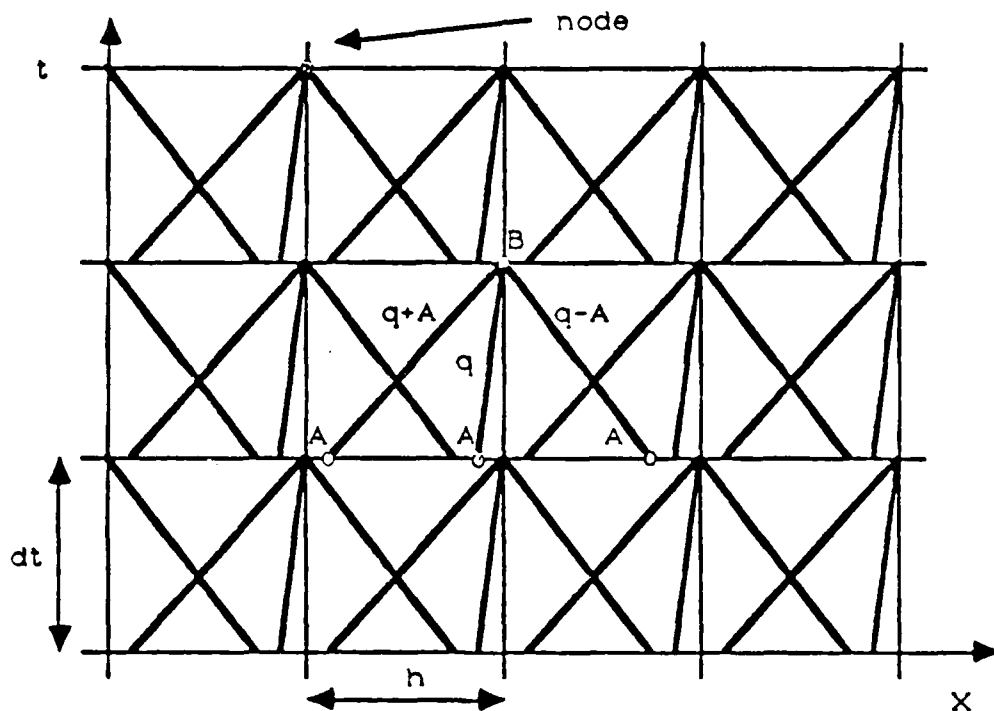


Figure 3.1 Overall Algorithm for QAZ1D Method

"BONDRY" subroutine is called and returns the updated values at those nodes. In between, the correct and most effective algorithm to update the node is determined. These algorithms are coded into eight different condition subroutines. These routines calculate the interpolated Riemann variables associated with the three characteristics ($q+A$, q , and $q-A$), plus the spatial derivatives at $\partial q/\partial s$ and $\partial A/\partial s$. This allows "SWEEP" to calculate $\Delta \bar{w}$ and \bar{z} . The integral of \bar{z} is then determined, and the update for the variables of that node in $\partial \bar{w}$ are computed. \bar{w} is stored until the entire mesh has been swept. Then the variable arrays (\bar{w}) are updated to the next time interval.

To choose the proper condition routine, the following information about conditions in the interval on either side of the node are expressed in two 3 digit integer variables, SHOCK and CNTACT. The value of these variables provide a code for the following information:

SHOCK--The existence of a shock within an interval, and, if so, the direction of travel, and if the shock will cross the node in front of it within the current time step.

CNTACT--The existence of a contact surface within an interval, and, if so, the direction of travel, and if the contact surface will cross the node in front of it within the current time step.

Figure 3.2 illustrates the regions around a node. Table V lists the values that SHOCK and CNTACT may have, and their meaning. Figures 3.3, 3.4, and 3.5 illustrate examples of what different SHOCK and CNTACT values mean. At each node

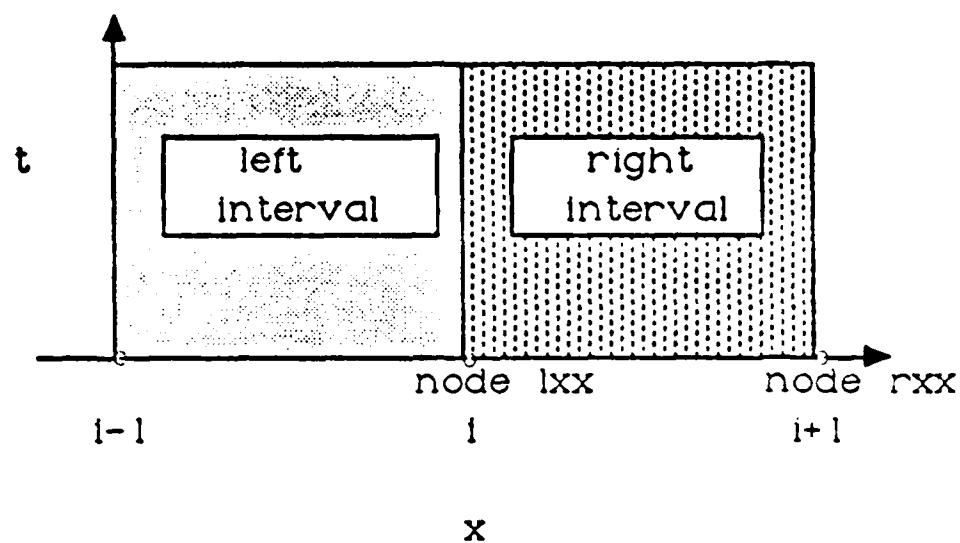


Figure 3.2 Interval and Node Description used in "SWEEP"

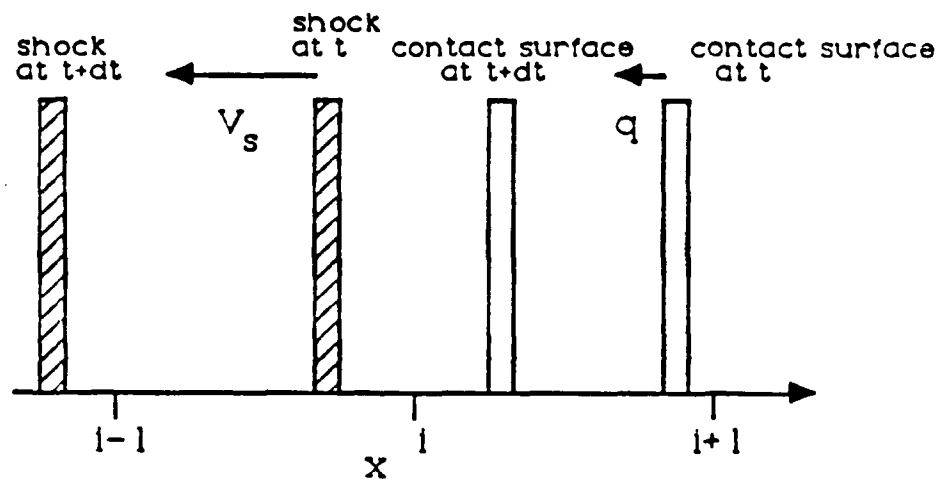


Figure 3.3 Schematic of SHOCK = 331 and CNTACT = 232

TABLE V
VALUES OF PARAMETERS SHOCK AND CNTACT

| SHOCK | | | |
|--------------|-----------------------------------|--------------------------------|--|
| <u>Value</u> | <u>Located in interval on</u> | <u>Direction of travel</u> | <u>Crosses node in path within time step</u> |
| 100 | No Shock | N/A | N/A |
| 321 | Left | Right | Yes |
| 322 | Left | Right | No |
| 331 | Left | Left | Yes |
| 332 | Left | Left | No |
| 221 | Right | Right | Yes |
| 222 | Right | Right | No |
| 231 | Right | Left | Yes |
| 232 | Right | Left | No |
| CNTACT | | | |
| 100 | No contact surface | N/A | N/A |
| 321 | Left | Right | Yes |
| 322 | Left | Right | No |
| 331 | Left | Left | Yes |
| 332 | Left | Left | No |
| 221 | Right | Right | Yes |
| 222 | Right | Right | No |
| 231 | Right | Left | Yes |
| 232 | Right | Left | No |

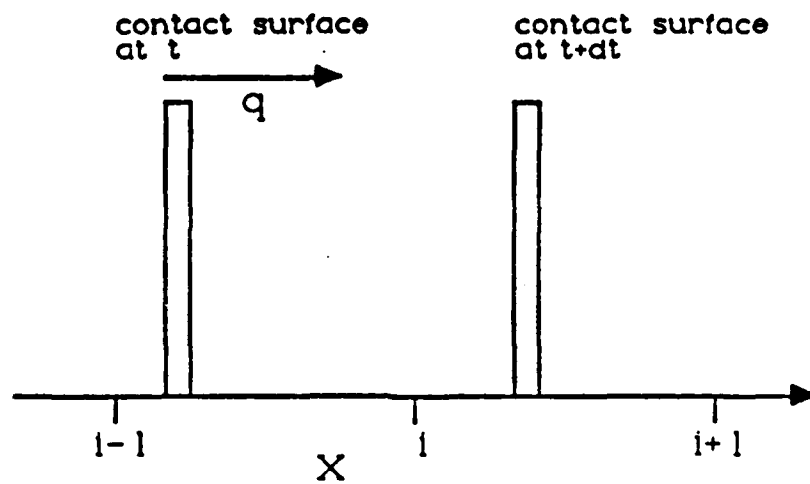


Figure 3.4 Schematic of SHOCK = 100 and CONTACT = 321

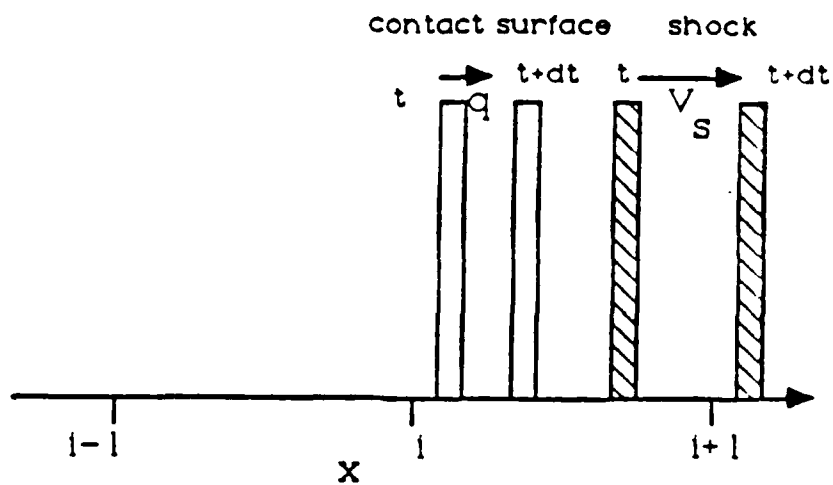


Figure 3.5 Schematic of SHOCK = 221 and CNTACT = 222

the code examines the shock and contact surface condition, along with relative location of the shock to a contact surface if both exist near the node, if the flow is supersonic or subsonic at the node, and the direction the flow is traveling. The appropriate algorithm is determined and called in the form of a condition subroutine.

The "SWEEP" routine is designed to handle all possible shock and contact surface combinations. Because the code does not solve the situation where the shock and contact surface intersect, as illustrated in Fig. 3.6, then any combinations after the shock and contact surface have crossed call condition routines that currently contain only messages. The code was intentionally prepared to be easily extended to treat the intersection of the shock and the contact surface, and beyond.

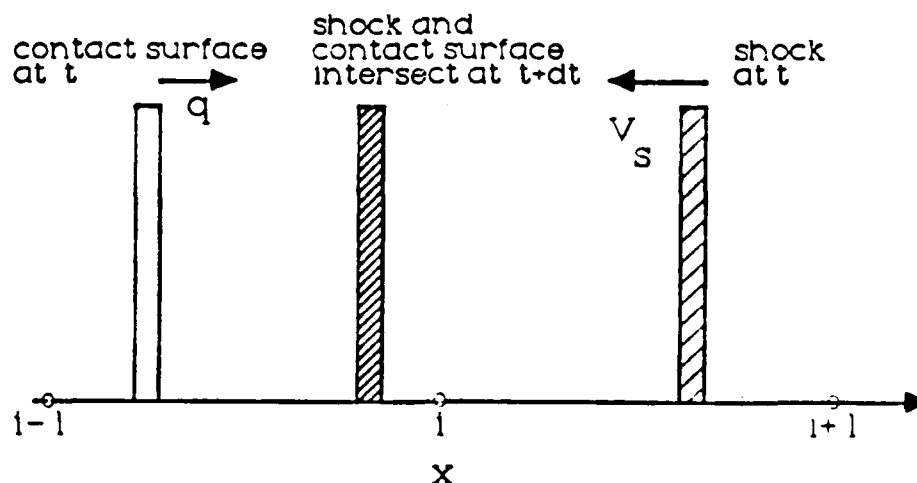


Figure 3.6 Schematic of SHOCK = 231 and CNTACT = 322

5. The "Condition" Subroutines

The ten subroutines that are called by "SWEEP" to calculate the interpolated values of the extended Riemann variables, R and Q , plus S , $\partial q/\partial s$, and $\partial A/\partial s$ are COND1, COND2, COND3, COND4, COND5, COND6, COND7, COND7S, COND7N, and COND8. The procedure used in COND1, COND2, COND3, and COND5 is that of Salacka [Ref. 2:pp. 36-37] modified to account for contact surfaces. Appendix C, Fig. C.5 is a general flowchart for these four routines. Essentially, an initial estimate is made of the characteristic slopes, λ , by enforcing the principle of domain of dependence. The assumption is made that the slopes are linear, and that the characteristics pass through point B, as defined in Fig. 3.7. Then q and A are computed at point A, which in turn yields a second estimate of the characteristic slope, λ .

The two slopes for each characteristic are compared. If they are not equal to within an acceptable error, the new estimate of λ is used to repeat the process until convergence is achieved. All three characteristics are handled simultaneously. The value of

$$\Delta s = \lambda \cdot \Delta t \quad (3.2)$$

is then determined for each characteristic $q+A$, q , and $q-A$. This allows linear interpolation of Q , R , and S at point A. The spatial derivatives, $\partial Q/\partial s$ and $\partial A/\partial s$ are computed from

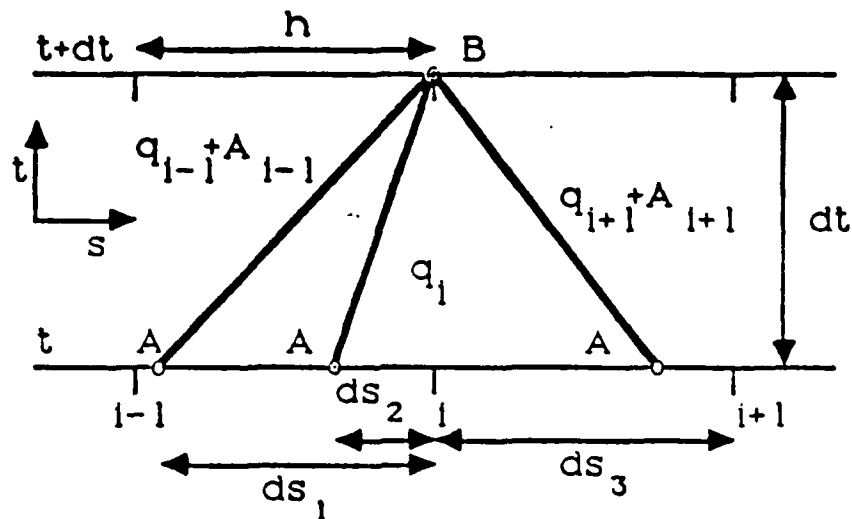


Figure 3.7 Computational Method for COND1 Routine

the values of q and A used in the estimate of the initial characteristic slopes.

The "COND1" subroutine is used when there are no contact surfaces or shocks within the interval on either side of the node, and flow is subsonic. The $q+A$ characteristic uses forward differencing, while the $q-A$ characteristic uses a backward differencing scheme to keep the initial domain of dependence of the numerical scheme outside the physical domain of dependence.

The "COND2" subroutine is a backward differencing algorithm used in situations when one or more discontinuities are in the right interval, or if flow is supersonic to the right. Fig. 3.8 illustrates the computational method

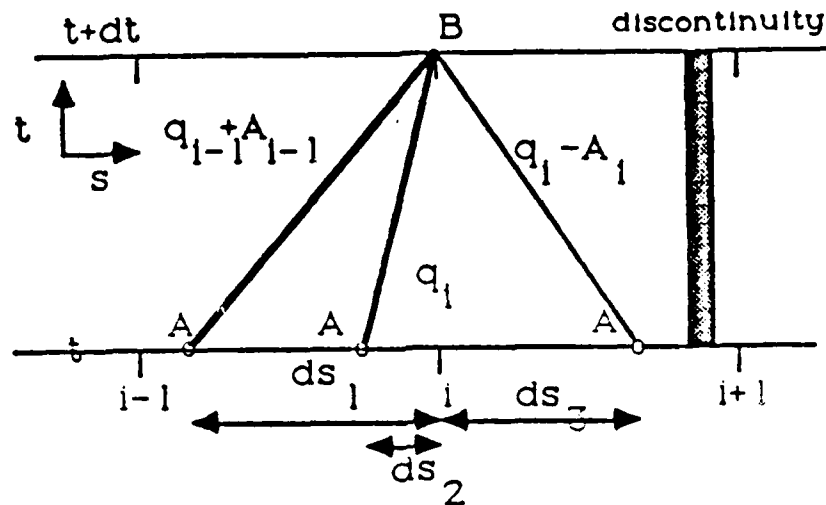


Figure 3.8 Computational Method for COND2 Routine

for "COND2". The characteristic slopes of $q+A$ and q are handled as in "COND1", but the $q-A$ characteristic slope is determined by a backward rather than a forward scheme. To interpolate between node i and $i+1$ would be incorrect because of the discontinuity in the values between them. The value of parameters in the right interval are interpolated from values in the left interval by assuming the derivatives do not change between the intervals up to the discontinuities (i.e., a shock and/or a contact surface).

The "COND3" subroutine is a forward differencing algorithm used in situations when one or more discontinuities are in the left interval, or if flow is supersonic traveling to the left. Figure 3.9 shows the computational

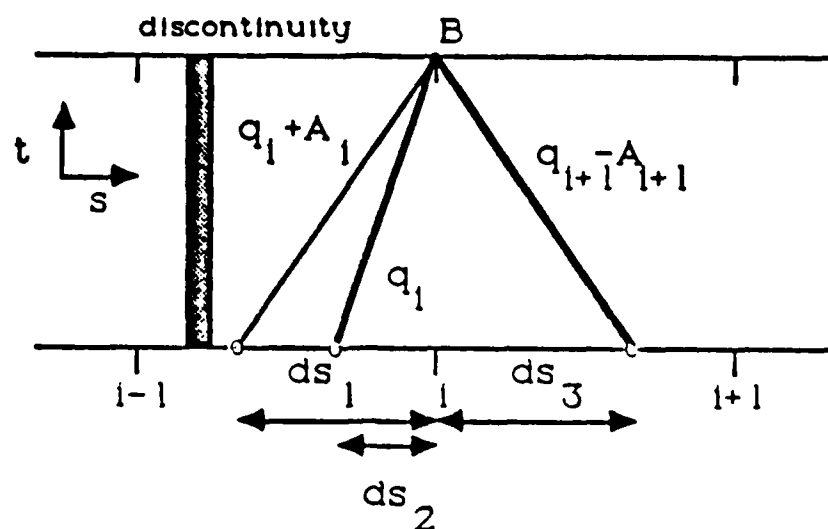


Figure 3.9 Computational Method for COND3 Routine

method for "COND3". The method is similar to that in COND2 but the $q+A$ characteristic slope has to be interpolated from values of node i instead of node $i-1$ using a forward difference scheme. The characteristic slope for $q-A$ and q are estimated by forward difference schemes using values of node $i+1$ and i respectively.

For conditions, such as that illustrated in Fig. 3.10, where a discontinuity is in the interval opposite the direction of supersonic flow then COND5 subroutine is called. A mix of COND2 and COND3 is used. For flow to the right the method of COND2 is implemented, while flow to the left is the same as for COND3.

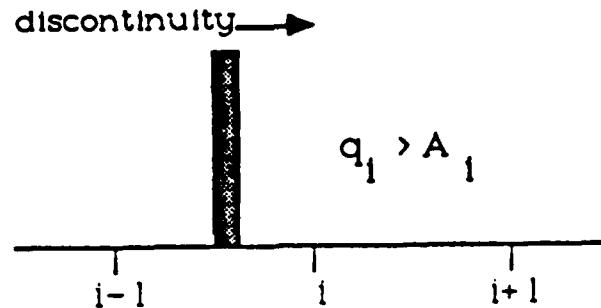


Figure 3.10 Example Condition for "COND5" Routine

Subroutine "COND4" is called whenever a node is to be crossed by a discontinuity from either direction. The flowchart for "COND4" is shown in Appendix C, Fig. C.6. Two integer vector arrays, LNODE and RNODE, are used to store the following information on the node being crossed.

- A) the node number, I
- B) the value of SHOCK
- C) the value of CONTACT
- D) the current value of the integer time step, J

The information is used in "CORRECT" to update the node to the next time level. In the current code the maximum number of nodes that can be crossed during any time step is two. Figure 3.11 shows an example of this situation, and defines the assignment of values. Because the nodes are swept from left to right, the left-most node is assigned to LNODE, while RNODE applies to any node crossed to the right of the LNODE node. The values of \bar{u} are set to zero, so

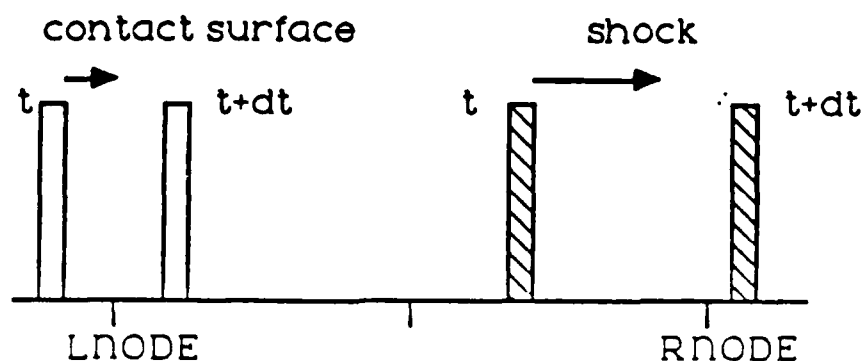


Figure 3.11 Example of "COND4" Routine Situation

that these nodes are skipped when updating occurs in "SWEEP". The main program interprets the value of J in LNODE and RNODE to determine if zero, one, or two nodes need to be corrected in "CORRECT". If zero nodes are crossed during a time step then "CORRECT" is not called.

The "COND6" and "COND8" subroutines are designed for future extensions of the entire code. Until the Riemann problem illustrated in Fig. 3.6 is coded, then any situation after the shock and contact surface intersect cannot be computed. However, "SWEEP" is already coded to call "COND6" for the situation illustrated in Fig. 3.12, or its mirror image. "COND8" would be called for all the other situations after the shock and contact surface interaction has taken place. Currently, both subroutines output messages on the screen describing the situation, and set HALT equal to 1 to terminate the program.

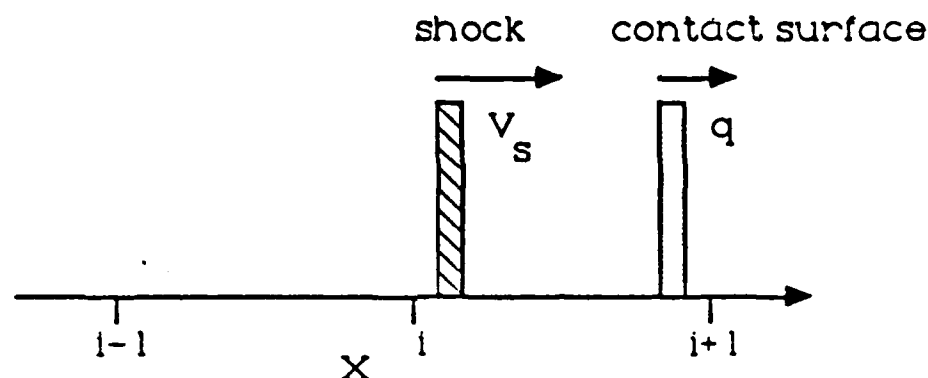


Figure 3.12 Example of "COND6" Situation

The "COND7", "COND7N", and "COND7S" subroutines are all related to each other. They are intended to contain code that would facilitate the shock and contact surface intersection and solve the resulting Riemann problem. The "COND7" routine is called when the contact surface and shock moving in opposite directions would cross in a time step within an interval. The routine would calculate when and where within the time and space domain the intersection would occur based on the known velocity of each discontinuity and their current locations. This new time could then be used to rerun "SWEEP" with the shock and contact surface exactly on top of each other at the end of the new time step. The "COND7N" routine is for the same situation, but when a node is crossed by either discontinuity during the same time step. The same procedure is done, with the requirement to ensure the node crossed is properly updated

by "CORRECT" during the new time step. The "COND7S" routine is called when a shock and contact surface are located at the same point at a time other than zero. The subroutine would contain the code to solve the Riemann problem set up by "COND7N" or "COND7" routines.

6. The "CORRECT" Routine

Appendix C, Fig. C.7 shows the flowchart for "CORRECT". The "CORRECT" routine corrects nodes that have been crossed by a discontinuity (i.e., a shock or a contact surface) or are straddled by both. The routine takes the information from LNODE and RNODE arrays to determine which node or nodes are to be updated, and if there is any shock and contact surface interaction at the nodes. Then, using subroutines "DELTAX", "SKJUMP", "CSJUMP", "EXTRAP", "INTERP", and "BBDY" the concepts from Section II are imposed to calculate the jump in the parameters R, Q, S, A and q at the node in question to the new time level.

7. The "BONDY" Routine

The "BONDY" routine computes the new values of the parameters Q, R, and S at the boundary nodes for time step t. The concepts of Section II for an open or closed boundary are coded as shown in the flowchart of "BONDY" given in Appendix C, Fig. C.8. The routine uses the "COND1", "COND2", or "COND3" routines to calculate the correct w, and then solves for w the same as in "SWEEP". If a shock crosses an open boundary node during a time step,

then "SKJUMP" routine is used to calculate the values behind the shock at the node.

8. The "SRFLCT" Routine

The "SRFLCT" subroutine is called to calculate the values of Q, R, S, q and A at the boundary node when a shock reflects from a solid boundary. Appendix C, Fig. C.9 shows the flowchart of "SRFLCT" which codes the analysis in Section II.D.2 for closed boundary shock reflection. The time for the shock to reach the solid boundary, ∂t_w , is computed. Then the excess time in the time step, ∂t , is given by

$$\partial t_{ex} = \partial t - \partial t_w \quad (3.3)$$

After the new shock velocity, V_s , is calculated, then the new location of the reflected shock is known from multiplying V_s by ∂t_{ex} and adding or subtracting from the boundary location.

9. The "DELTAX" Routine

The "DELTAX" subroutine is used in "CORRCT" and "BONDRY" to calculate the location within an interval of a discontinuity in terms of x. Figure 3.13 defines the various displacements which are calculated.

10. The "SKJUMP" Routine

The "SKJUMP" subroutine computes the conditions downstream of a stationary normal shock as described in

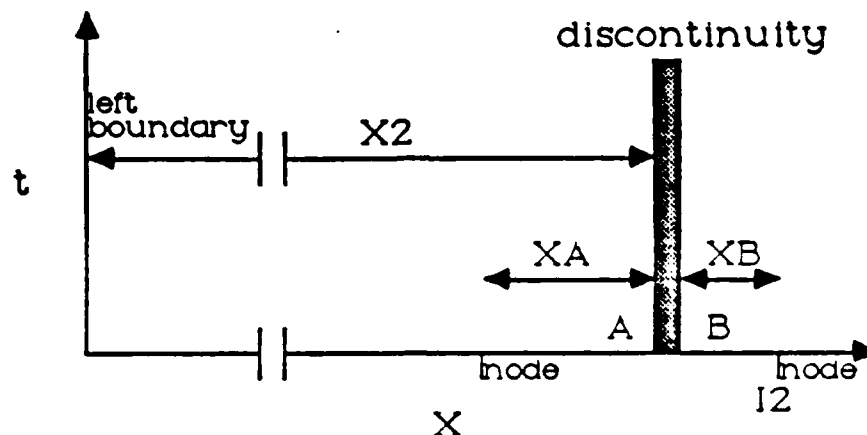


Figure 3.13 Discontinuity Location within an Interval

Section II.D.2. The "CORRCT", "TRAK", and "BONDRY" subroutines call "SKJUMP" when required, to obtain the speed of sound (A), entropy (S), and velocity (q) behind the shock.

11. The "CSJUMP" Routine

The "CSJUMP" subroutine computes the velocity and speed of sound change across a contact surface as described in Section II.D.3. The "CORRCT" and "TRAK" routines call "CSJUMP".

12. Numerical Support Routines

The "EXTRAP", "INTERP", and "BBDRY" routines are used by "CORRCT", "BONDRY", "TRAK", and "DBURST" to extrapolate or interpolate data to the face of a discontinuity or point. In particular, "BBDRY" is an interpolation routine used within an interval near a boundary.

13. The Output Routines

The four output subroutines used are those developed by Salacka [Ref. 2:pp. 39-40]. The "BORDER" and "PLOT" routines output plots of pressure, velocity, density, and modified entropy distributions versus a non-dimensionalized tube length at preset time intervals. "EXACT" creates a plot of the exact density distribution at selected points and compares it with the computed density distribution for a selected time step. The "LIST" routine outputs to files 8, 9, and 10 tabular listing of computed data. The files are sent to the user's permanent storage disk.

A value of GRAPHS equal to one causes "BORDER" to be called in the main program. This defines plot axis, labels, and headings. "PLOT" is then called once at time zero, and at every time step set by SKIP.

If GRAPHS is set equal to two, then "EXACT" is called every SKIP time steps to plot six exact density values and the computed density distribution. The six exact points are:

- A) The two boundary points
- B) The head and tail of the rarefaction wave
- C) A point just behind the contact surface
- D) A point just behind the shock.

The location of the exact values is based on elapsed time and known exact values for wave velocities entered with the initial conditions for the problem.

"LIST" routine is called every SKIP time steps when GRAPHS is equal to zero. File 9 contains a tabular listing of the Riemann variables, modified entropy, pressure, density and velocity distributions, elapsed time, discontinuity velocity and location. File 10 is a tabular listing of the location of the shock, contact surface, and the head and tail of the expansion wave computed at every SKIP time steps. File 8 contains the exact values of the location of the shock, contact surface, and the head and tail of the expansion wave.

IV. RESULTS

Four test cases were run using the E1DV2 code. The purpose was to verify the ability of the code to determine the unsteady flow process and correctly simulate varying boundary conditions and flow directions in the Riemann shock tube problem. The shock tube is illustrated in Fig. 4.1 with the high pressure on the left. The tube is divided into two sections by a diaphragm. When the diaphragm is burst, the pressure equalizes through a shock wave traveling into the expansion chamber, and an expansion or rarefaction

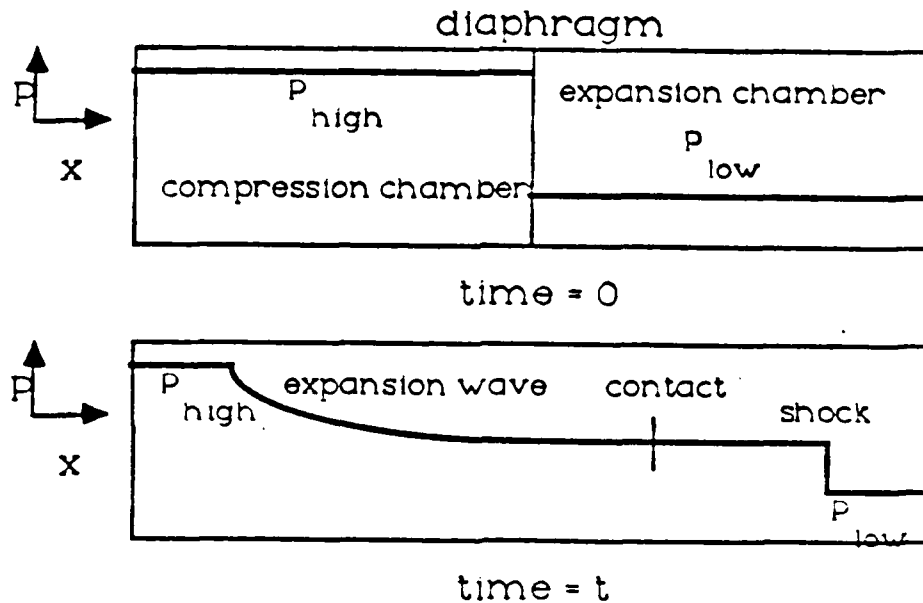


Figure 4.1 Shock Tube at $t = 0$ and $t = t$

wave traveling into the compression chamber. A contact discontinuity is behind the shock wave, traveling at the particle velocity [Ref. 6:p. 30].

A. TEST CASE 1

Test Case 1 was designed to demonstrate shock reflection and expansion wave reflection at solid boundaries. Test Case 1 had the following initial and boundary conditions:

- 1) Pressure and density ratios equal to five, with high pressure on the left
- 2) Temperature ratio equal to unity
- 3) Both boundaries were closed
- 4) Diaphragm was located at $x = 0.5$
- 5) Computational mesh had 101 nodes
- 6) Maximum time step, JSTOP, = 109, with SKIP = 18.

Plots of the results obtained for the pressure, density, velocity, and modified entropy distributions are shown in Fig. 4.2 and Fig. 4.3. Fig. 4.2 shows the computation up to time step 55, while Fig. 4.3 takes the computation from time step 56 to termination. The output of "EXACT", a plot of the exact density compared with the computed density distribution is shown in Fig. 4.4. Fig. 4.5 illustrates the spatial location versus time for exact and computed shock, contact surface, and head and tail of the expansion wave. Data for Fig. 4.5 were taken from file 10 and file 8. Fig. 4.6 shows plots of pressure, density, modified entropy, and velocity distributions for the same initial and boundary

SHOCK TUBE RESULTS

FIRST ORDER N -101
 DENSITY RATIO - 5.0 TEMP RATIO - 1
 PRESSURE RATIO - 5.0

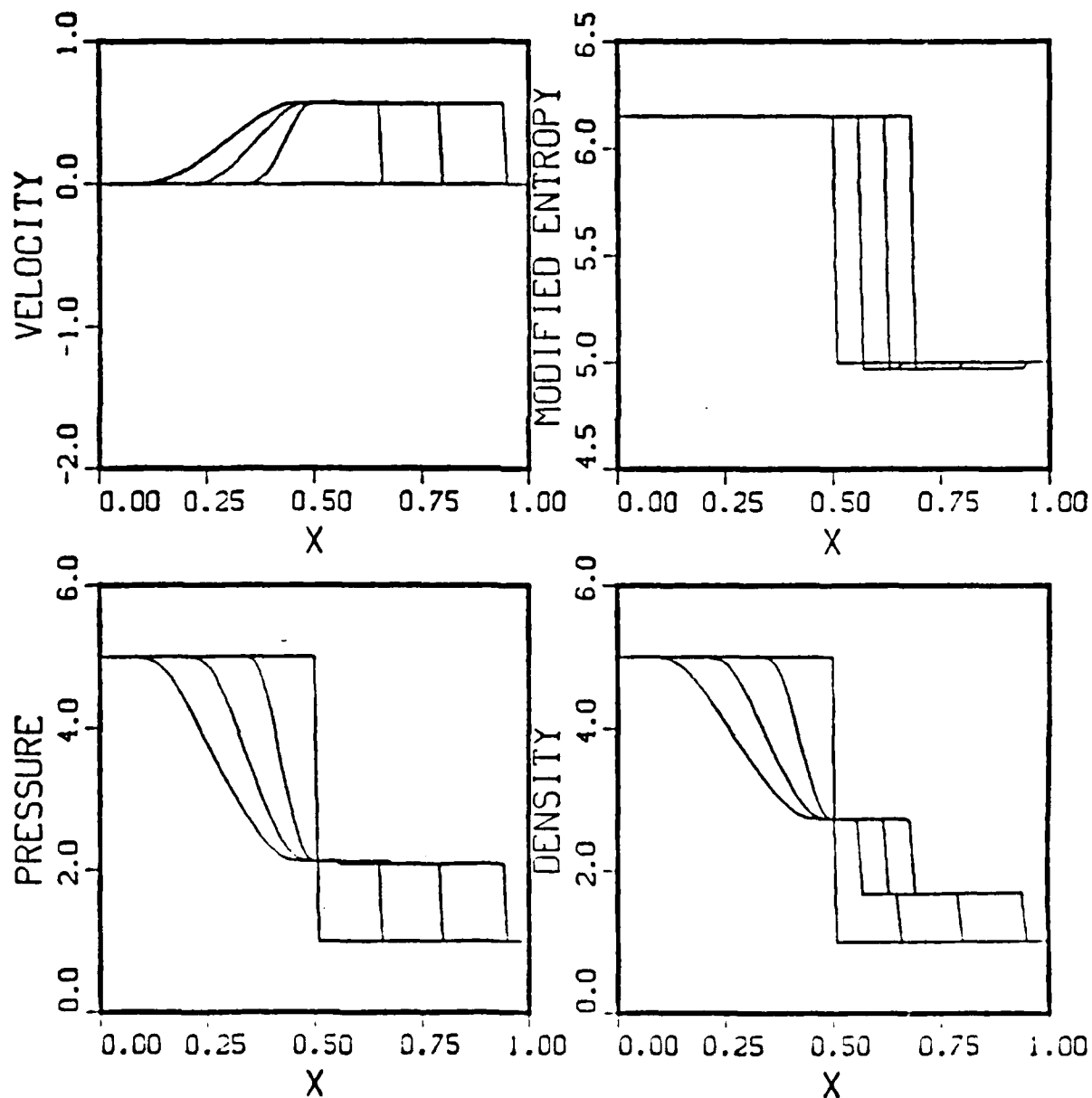


Figure 4.2 Test Case 1 ($J = 1$ to $J = 55$)

SHOCK TUBE RESULTS

FIRST ORDER N - 101
 DENSITY RATIO - 5.0 TEMP RATIO - 1
 PRESSURE RATIO - 5.0

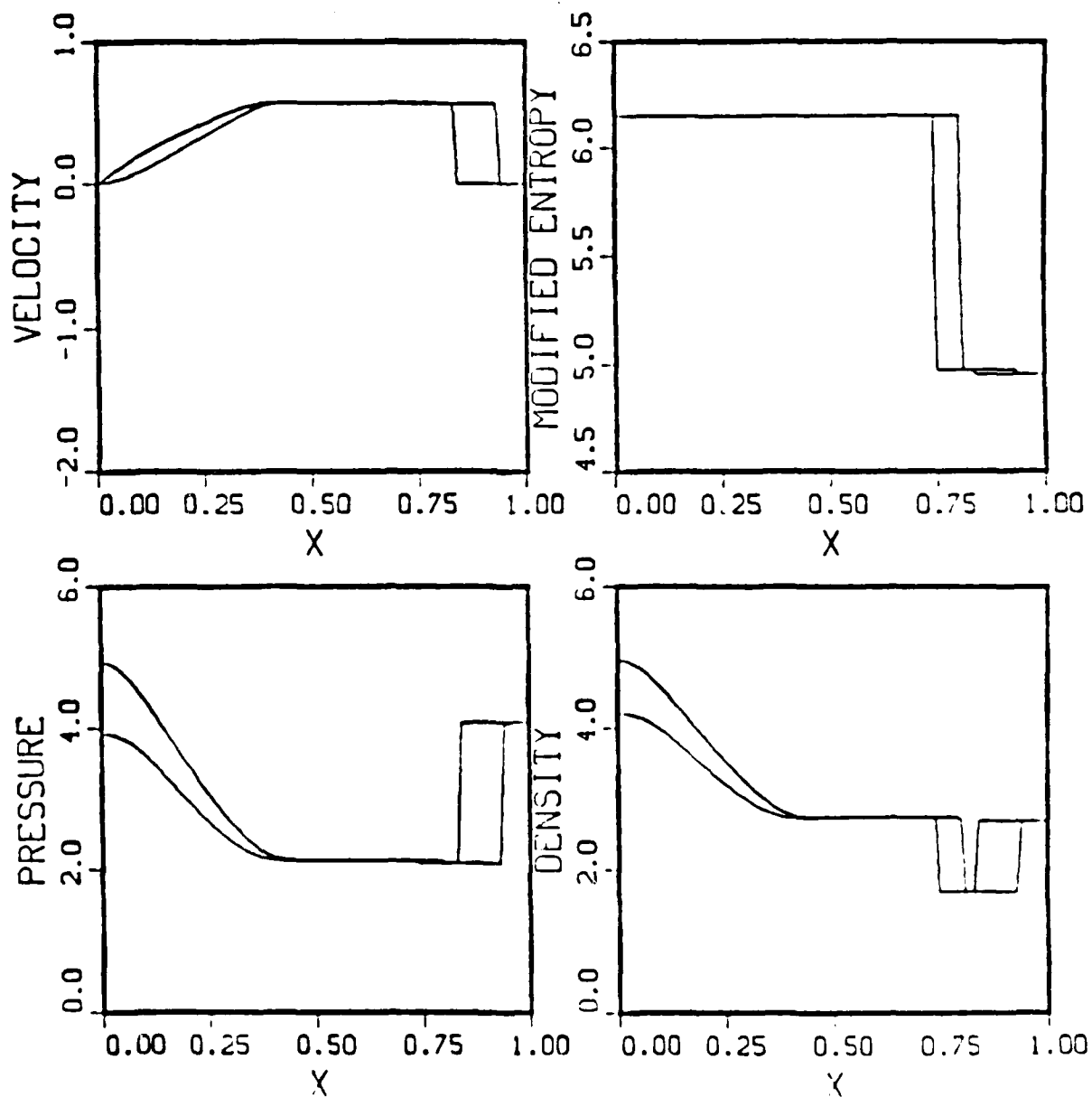


Figure 4.3 Test Case 1 (J = 56 to J = 109)

DENSITY DISTRIBUTION
 FIRST ORDER N - 101
 DENSITY RATIO - 5 TEMP RATIO - 1
 PRESSURE RATIO - 5

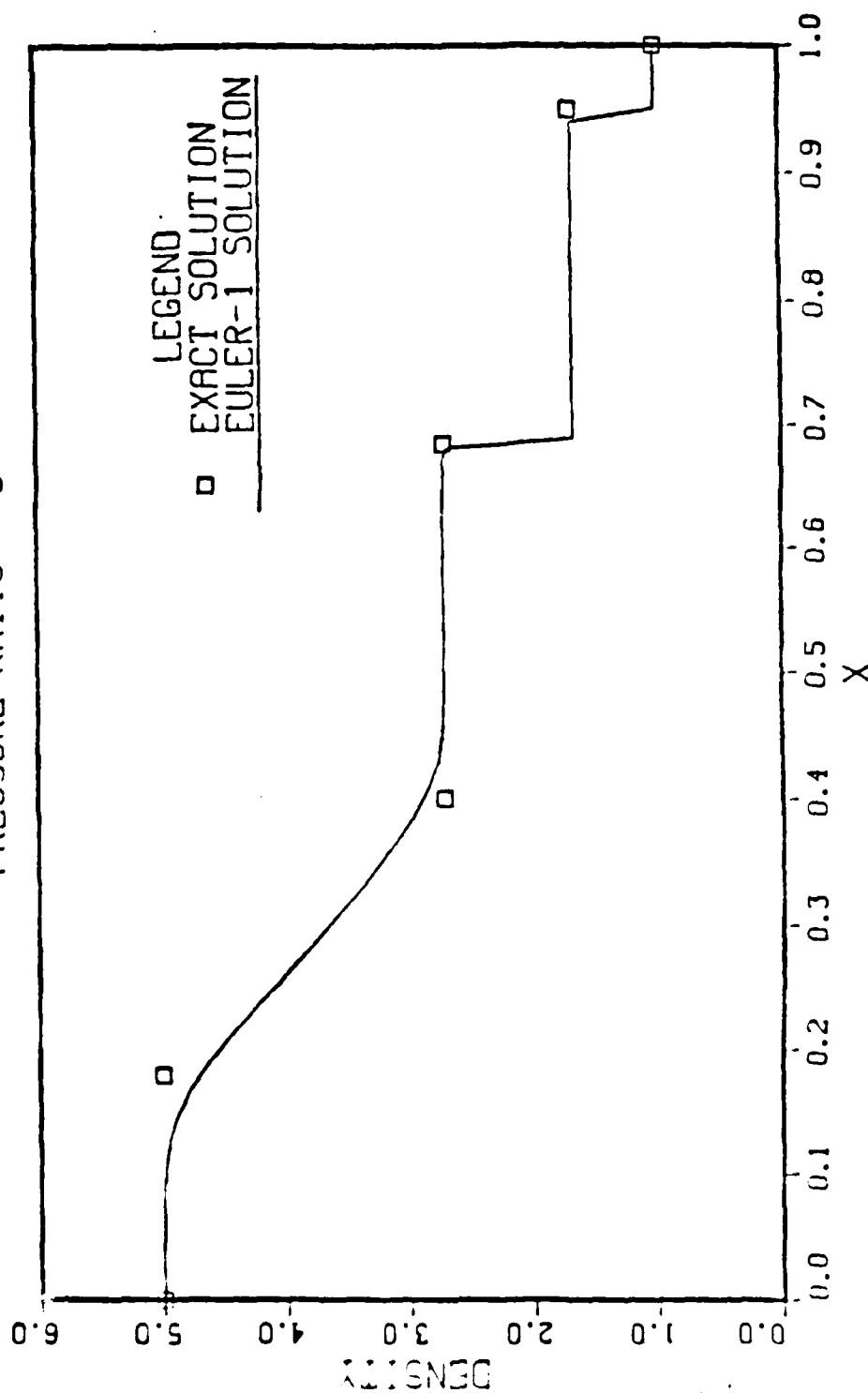


Figure 4.4 Exact and Computed Density Distributions

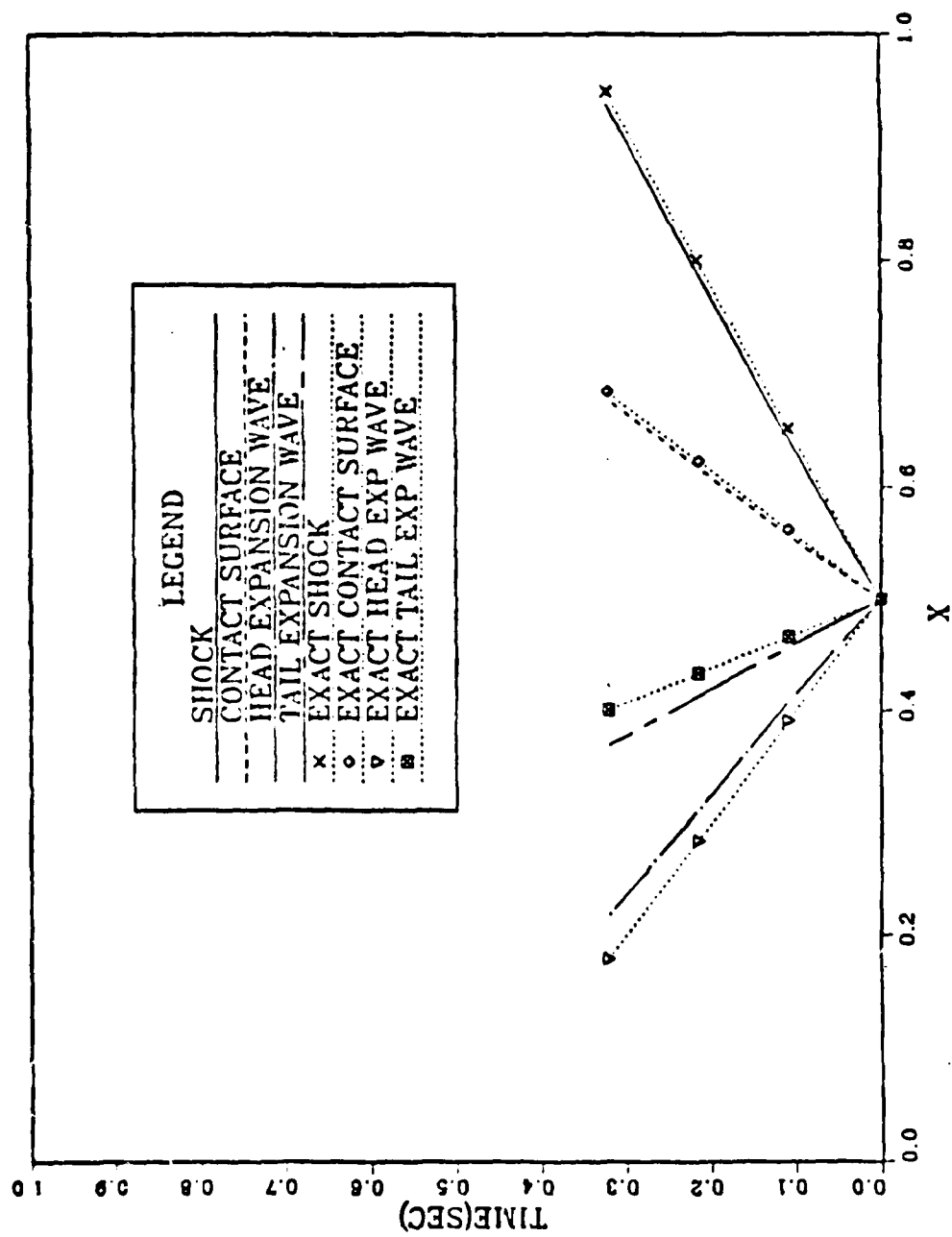


Figure 4.5 Location of Discontinuities versus Time (Test Case 1)

SHOCK TUBE RESULTS

FIRST ORDER N -101
 DENSITY RATIO - 5.0 TEMP RATIO - 1
 PRESSURE RATIO - 5.0

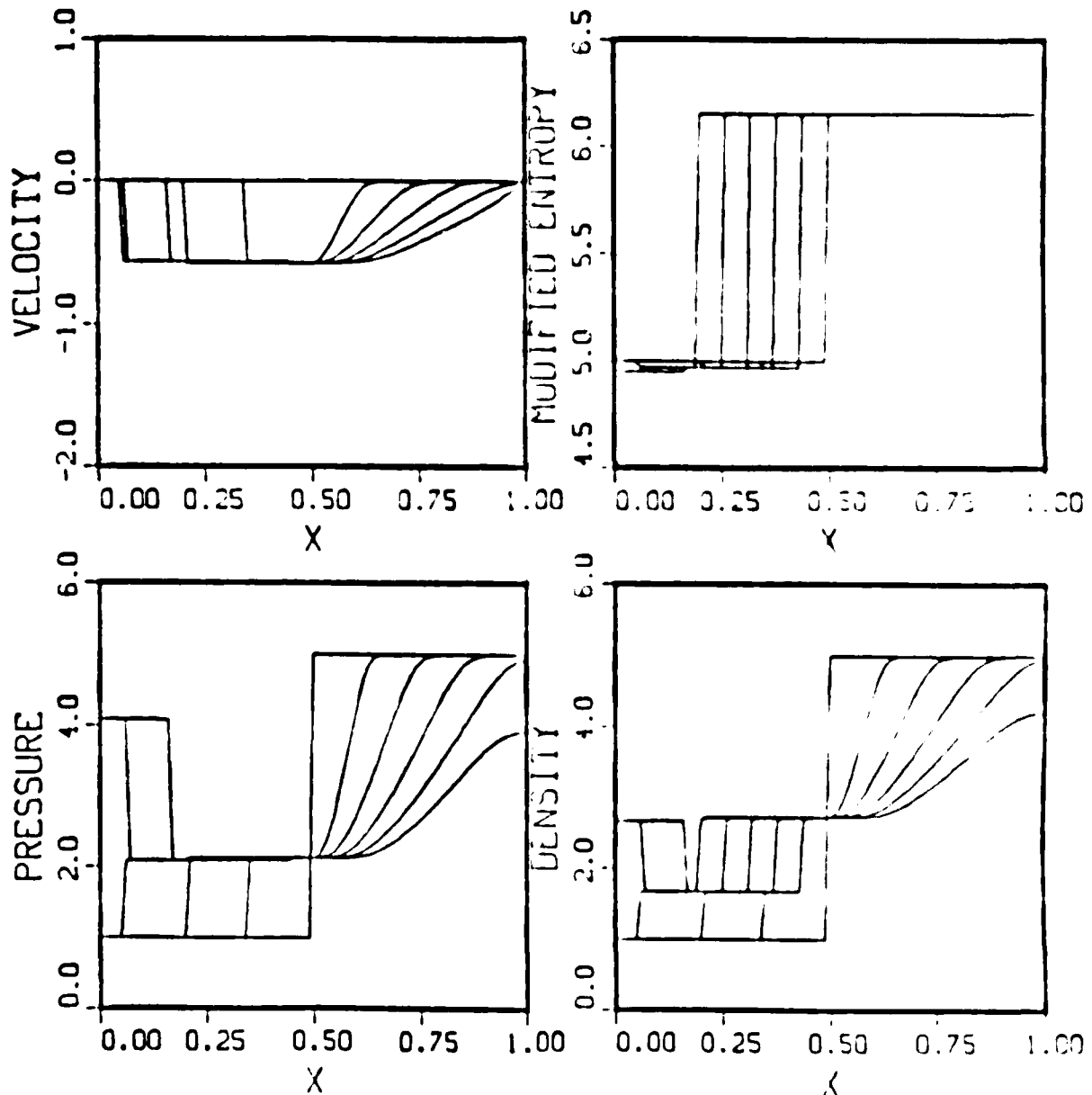


Figure 4.6 Test Case 1, High Pressure on Right

conditions but with high pressure on the right. Figure 4.6 includes data corresponding to those in both Fig. 4.2 and Fig. 4.3 for the high pressure initially on the left.

B. TEST CASE 2

Test Case 2 was to demonstrate an open boundary, constant pressure, expansion wave interaction. The test case was run with the following initial and boundary conditions:

1. Pressure and density ratios equal to 5.0, with high pressure on the left
2. Temperature ratio equal to unity
3. The left boundary was open, with a constant pressure ratio across the boundary of 4.5
4. The right boundary was open, with an adjustable pressure ratio that in effect extended the length of the tube
5. The diaphragm was located at $x = 0.74$
6. Computational mesh had 51 nodes
7. Maximum time step, $JSTOP = 10$, $SKIP = 2$.

Plots of the results for the pressure, density, velocity, and modified entropy distributions are shown in Fig. 4.7. The test case was rerun with high pressure on the right. Figure 4.8 shows the results for pressure, density, and temperature ratios the same. The diaphragm was then at $x = 0.26$. Boundary conditions were reversed. The same computational mesh, and output control were used.

SHOCK TUBE RESULTS

FIRST ORDER $N = 51$
 DENSITY RATIO - 5.0 TEMP RATIO - 1
 PRESSURE RATIO - 5.0

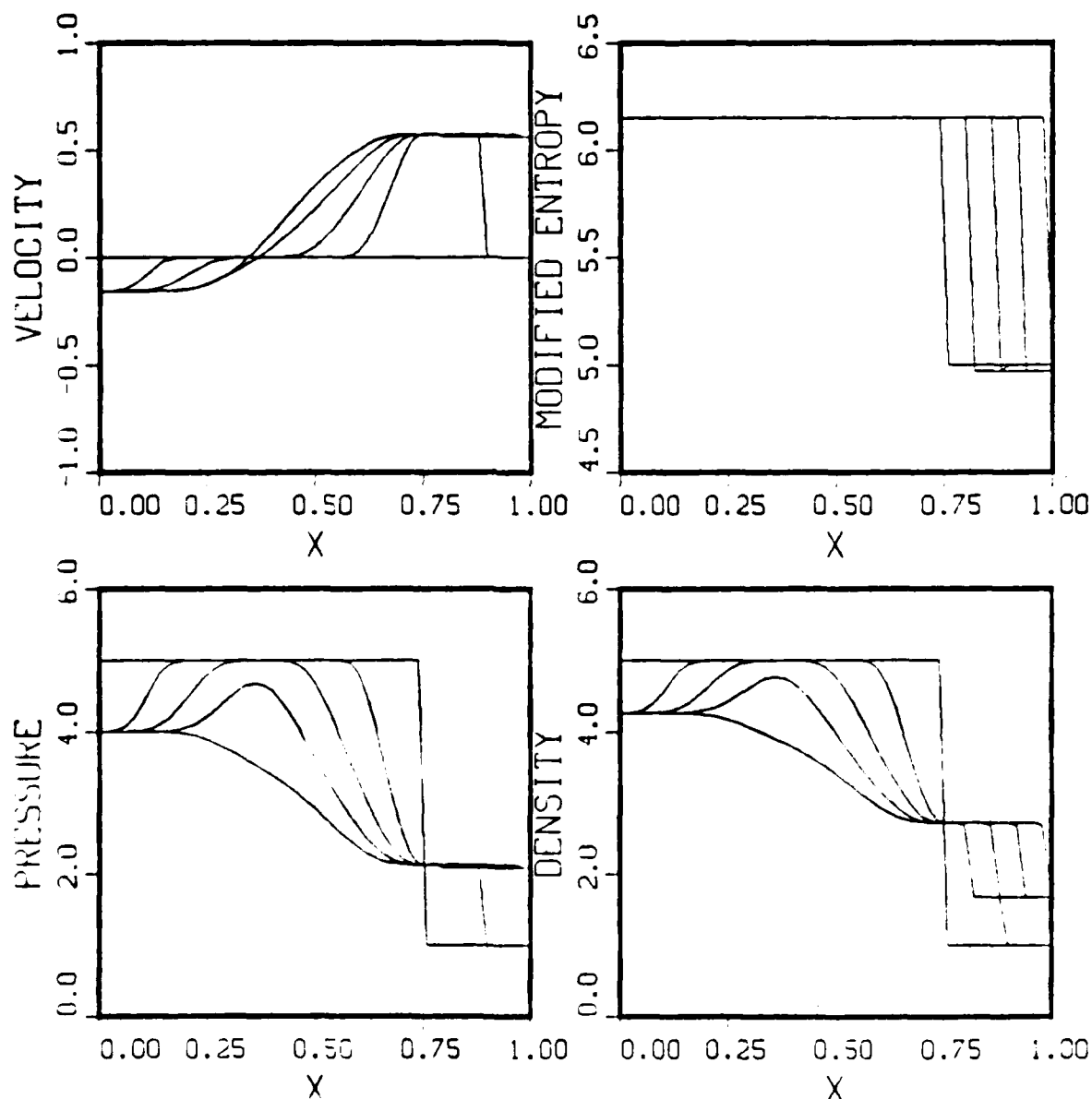


Figure 4.7 Test Case 2, High Pressure on Left

SHOCK TUBE RESULTS

FIRST ORDER N - 51
 DENSITY RATIO - 5.0 TEMP RATIO - 1
 PRESSURE RATIO - 5.0

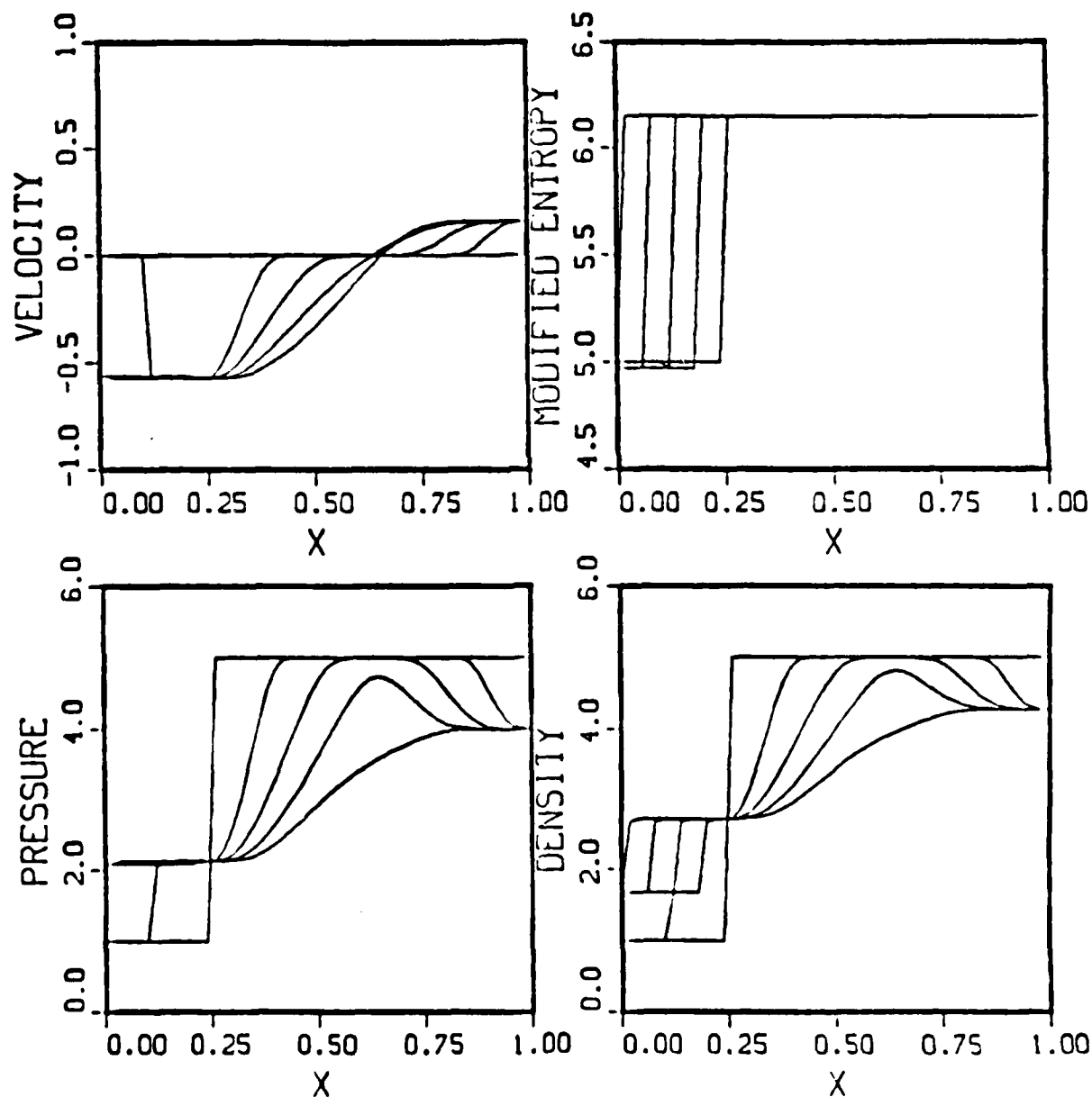


Figure 4.8 Test Case 2, High Pressure on Right

C. TEST CASE 3

Test Case 3 was designed to demonstrate a shock exiting an open boundary with constant pressure maintained at the boundary itself. The following initial and boundary conditions were set:

- 1) Pressure and density ratios equal to 5.5, with high pressure on the left
- 2) Temperature ratio equal to unity
- 3) Left boundary was closed
- 4) Right boundary was open with a constant pressure ratio across the boundary of unity
- 5) Diaphragm was located at $x = 0.5$
- 6) Computational mesh had 101 nodes
- 7) Maximum time step, JSTOP, = 73, with SKIP = 18.

Figure 4.9 shows plots of the results obtained for pressure, density, modified entropy, and velocity distributions. Similarly, Fig. 4.10 shows results obtained by putting the high pressure on the right, reversing the boundary conditions, and holding everything else the same.

D. TEST CASE 4

Test Case 4 was designed to demonstrate a lower pressure ratio, with shock wave reflection. The initial and boundary conditions were set as follows:

- 1) Pressure and density ratios equal to 3.2, with high pressure on the left
- 2) Temperature ratio equal to unity
- 3) Both boundaries were closed

SHOCK TUBE RESULTS

FIRST ORDER N - 101
 DENSITY RATIO - 5.5 TEMP RATIO - 1
 PRESSURE RATIO - 5.5

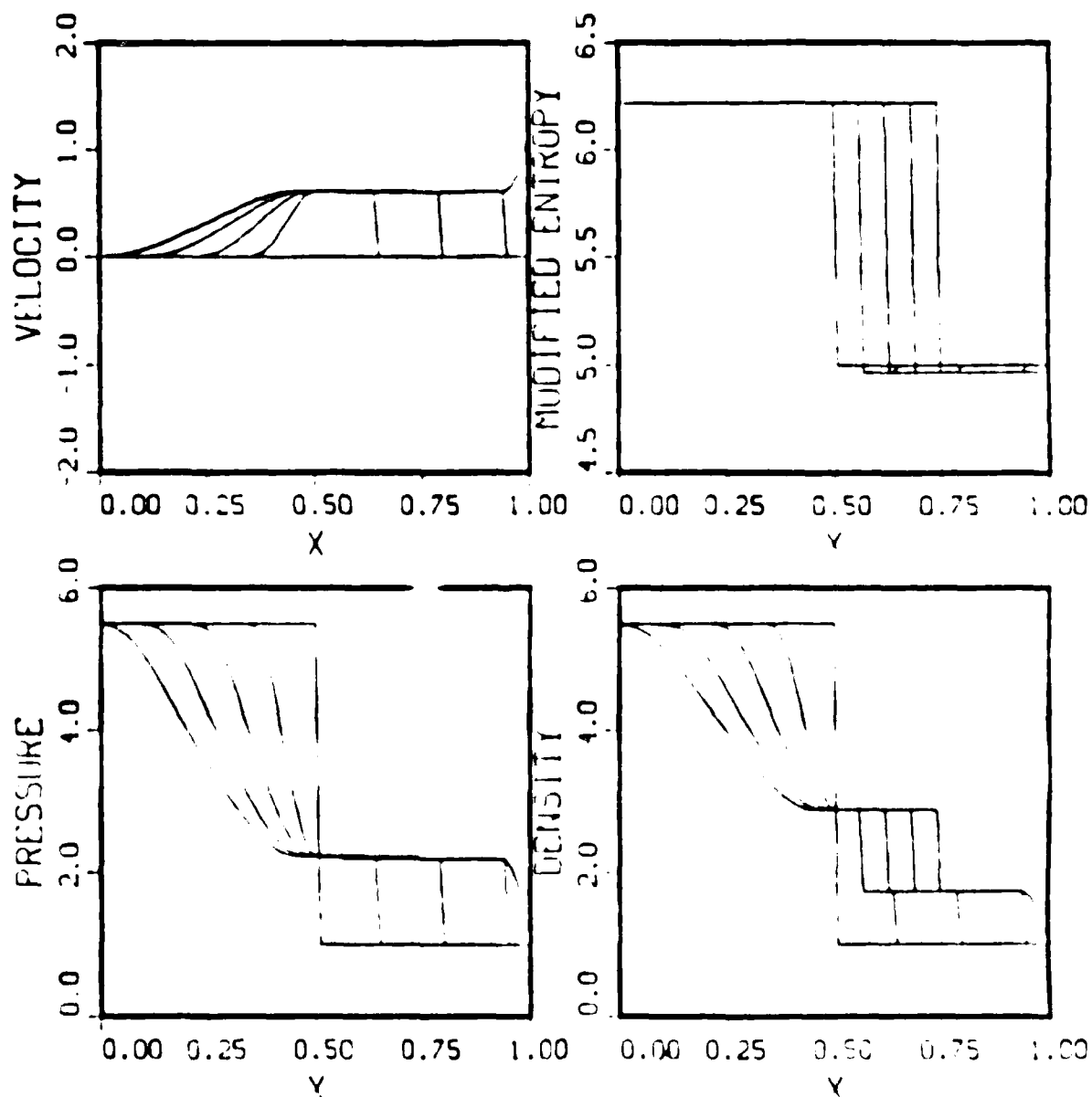


Figure 4.9 Test Case 3, High Pressure on Left

SHOCK TUBE RESULTS

FIRST ORDER N - 101
 DENSITY RATIO - 5.5 TEMP RATIO - 1
 PRESSURE RATIO - 5.5

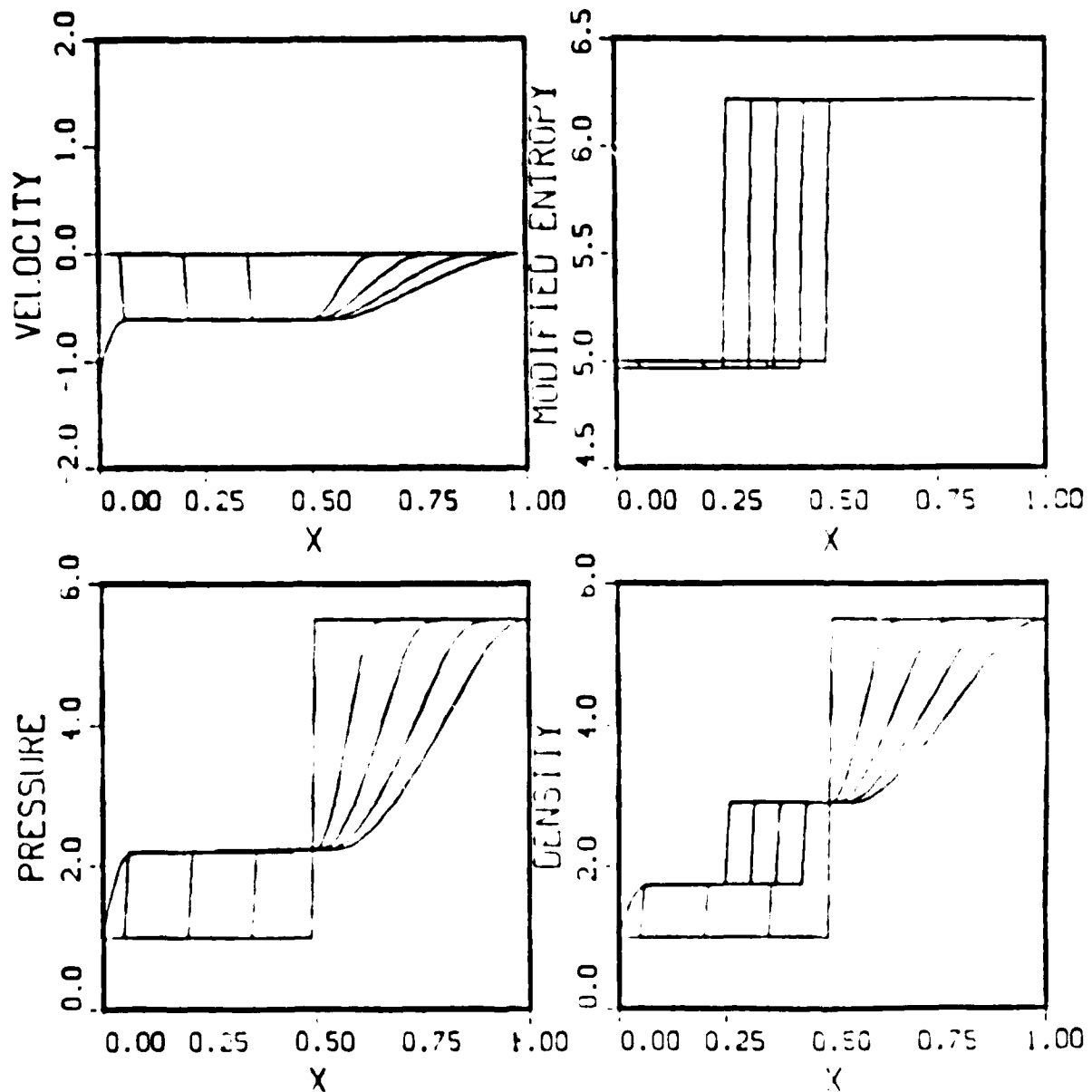


Figure 4.10 Test Case 3, High Pressure on Right

- 4) Diaphragm was located at $x = 0.5$
- 5) Computational mesh had 901 nodes
- 6) Maximum time step, JSTOP, = 601, with SKIP = 150.

Plots of the results obtained for the pressure, density, modified entropy, and velocity distributions are given in Fig. 4.11. Figure 4.12 shows the results obtained with the high pressure to the right, SKIP = 200, and holding all other parameters constant.

SHOCK TUBE RESULTS

FIRST ORDER N -901
 DENSITY RATIO - 3.2 TEMP RATIO - 1
 PRESSURE RATIO - 3.2

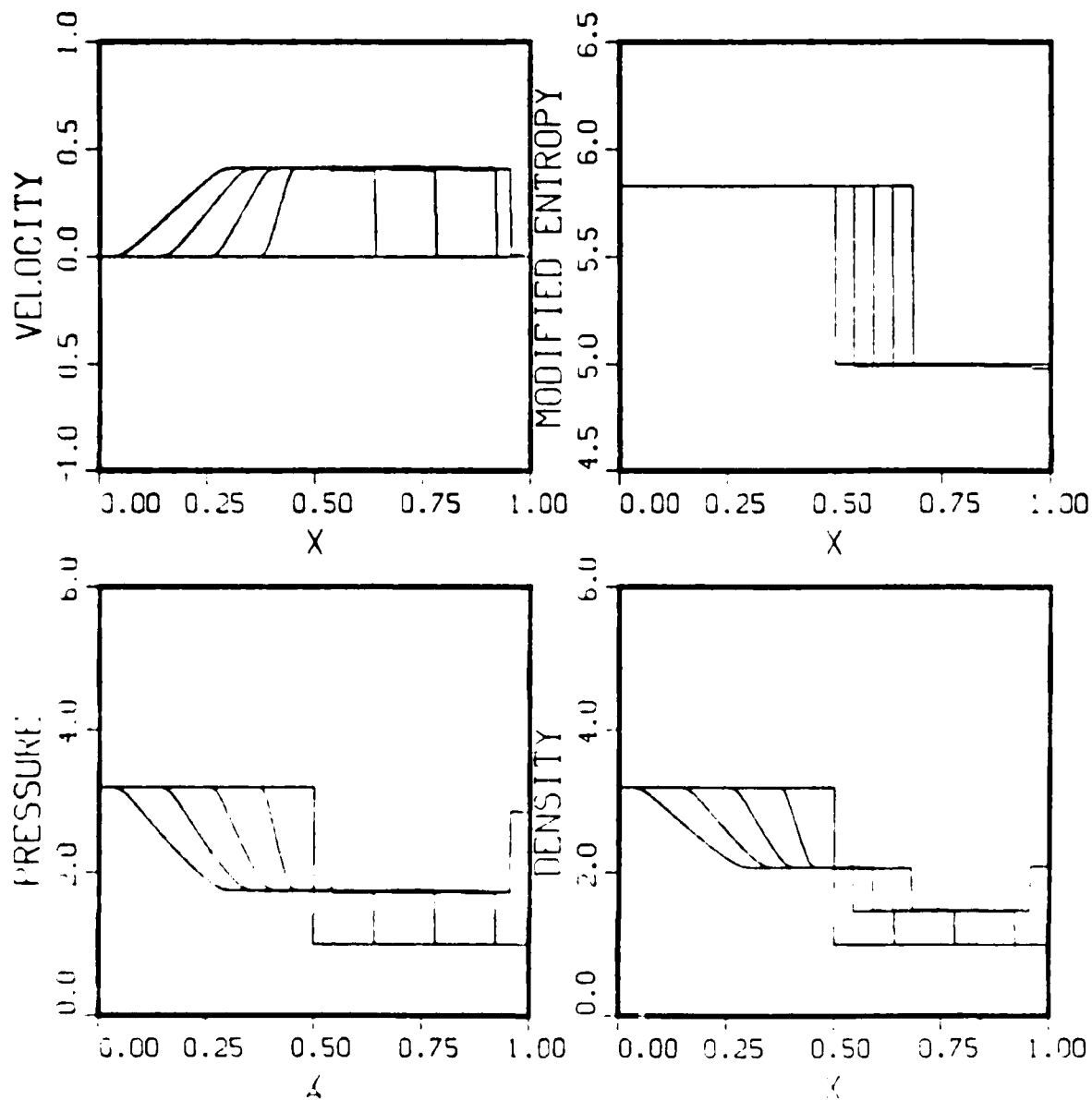


Figure 4.11 Test Case 4, High Pressure on Left

SHOCK TUBE RESULTS

FIRST ORDER N - 901
 DENSITY RATIO - 3.2 TEMP RATIO - 1
 PRESSURE RATIO = 3.2

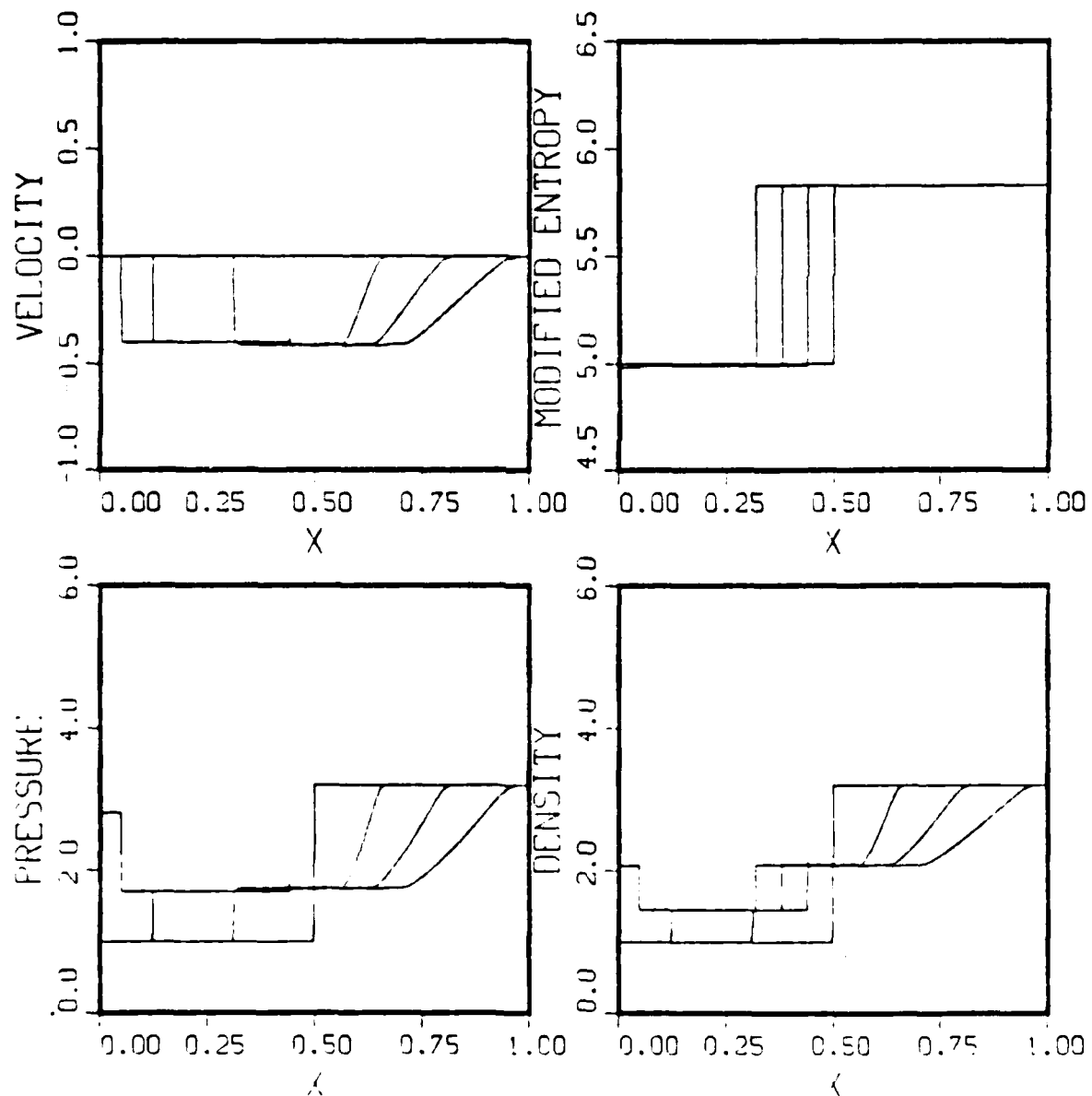


Figure 4.12 Test Case 4, High Pressure on Right

V. DISCUSSION

A. RESULTS OF TEST CASES

Overall, in the four specific test cases which were run, the expected qualitative flow behavior was produced by the code. Quantitatively, either experimental data or further exact solutions are needed to fully validate the computations. The results of each test case will be discussed separately.

Test Case 1 results in Fig. 4.2 show the formation of a well-defined shock wave traveling to the right, with the contact surface crisply defined following along behind. The entropy drops sharply across the shock and remains constant to the contact surface and then jumps to a steady, constant value to the left boundary as expected. Pressure and velocity remain constant through the contact surface. Velocity is positive since flow is to the right. The expansion wave is smeared from head to tail over the correct range. Figure 4.3 is a continuation of the flow problem with a sequential display of the results. The entropy continues to drop as the reflected shock travels back toward the contact surface. Notice that the velocity behind the shock is zero. At the left boundary as the head of the expansion wave reflects, pressure and density continues to drop while velocity is zero at the boundary. The density

distribution shows that the contact surface and shock are about to cross. The program terminated and issued a message before the next output call was made, when the code determined that the contact surface and shock would cross. The exact density distribution compared with the computed density distribution in Fig. 4.4 clearly shows the shock and contact surface are modeled very accurately with a medium mesh. The expansion process, consisting of infinitesimal changes propagating along the characteristics is fairly well modeled. Figure 4.5 shows that tracking of the contact surface and shock wave, in comparison with exact locations at set times, is excellent. Using $q+A$ and $q-A$ based on conditions behind the burst diaphragm as estimates of the velocities of the head and tail of the expansion wave respectively results in positions of the head and tail slightly different from those of an exact Riemann solver code [Ref. 3]. The exact code is based on a method given in [Ref. 9:pp. 181-191]. However, an examination of the tabulated output showed that the solution method predicted changes in the gradients to occur at similar locations to those computed using the $q+A$ and $q-A$ estimates. Consequently, while there may be some inaccuracy in the numerical solution, the method of tracking the head and tail of the expansion wave is acceptable.

The ability to reproduce the same conditions but with flow to the left is demonstrated clearly in Fig. 4.6. The

entropy, density, velocity, and pressure distributions are mirror-images of those for the case of high pressure on the left.

Test Case 2 results in Fig. 4.7 show that at time zero with a pressure ratio at the boundary of 4, which is lower than the preset value inside the tube of 5, an expansion wave traveling inwards is generated. Outflow is seen in the negative velocity distribution developing near the left boundary, where negative velocity implies flow traveling to the left. The contact surface and shock are clearly formed to the right of the off-center diaphragm, and travel to the right. The expansion wave generated at the diaphragm travels to the left and intersects with the expansion wave generated at the left boundary. The pressure distribution shows the propagation of these waves as the pressure drops behind the head of each wave. With conditions reversed, so that the high pressure is to the right of the diaphragm, the results in Fig. 4.8 are a mirror-image with velocity negative for flow to the left, and positive for the outflow to the right.

Test Case 3 results in Fig. 4.9 demonstrate that the shock wave meeting an open boundary where the pressure is held constant, is correctly computed to exit the tube. The density distribution shows the jump increase across the shock, then another jump increase across the contact surface. Then density plunges down as an expansion wave

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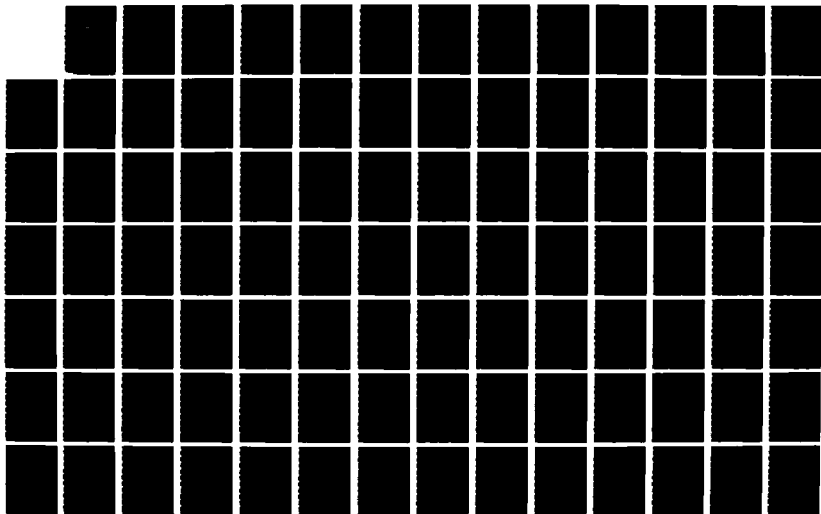
FURTHER DEVELOPMENT OF A ONE-DIMENSIONAL UNSTEADY EULER
CODE FOR WAVE ROTOR APPLICATIONS(U) NAVAL POSTGRADUATE
SCHOOL MONTEREY CA D T JOHNSTON MAR 87 NPS67-87-002

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MICROCOPY RESOLUTION TEST CHART

travels back into the tube when the shock exits. The pressure ratio at the open right boundary was held at unity, as seen in the pressure distribution which, at the boundary behaves similarly to the density distribution. The pressure at the boundary would be required to increase as the shock passed by, however the enforcement of the constant pressure locally causes an expansion wave to form traveling to the left. The entropy remains constant, at the value behind the shock, from the boundary back to the contact surface. There is a jump in entropy across the contact surface. The velocity distribution shows that as the shock travels to the right, the velocity jumps up. When the expansion wave generated at the right boundary travels inward the velocity continues to increase in magnitude while flowing to the right. The expansion wave generated at the diaphragm can be seen traveling to the left in the plots of pressure, density, and velocity. Reversing the situation and computing conditions for flow to the left, in Fig. 4.10, results in a mirror-image in the pressure, density, and entropy distributions. The velocity becomes negative since flow is to the left.

Test Case 4 is a repeat of the first test case but with a pressure ratio of 3.2 and a fine computational mesh. In Fig. 4.11 the density steps up across the shock and contact surface as they travel to the right. When the shock reflects the density jumps again. The velocity distribution

shows that the velocity drops to zero behind the reflected shock, after it had jumped up across the shock when it was traveling to the right. The instability seen in the velocity and pressure distributions near the midpoint occurred during the first 100 time steps. A numerical instability appears to generate at the diaphragm at time zero for pressure ratios less than about 3.0 which can be severe. For the conditions in this test case the transient instability damped out after the first 100 time steps. The shock, contact surface, and expansion wave are nevertheless seen to be accurately modeled. Figure 4.12 demonstrates that reversed conditions result in mirror-images of the computed conditions.

B. CURRENT LIMITATIONS OF THE CODE E1DV2

The current limitations of the E1DV2 code for modeling quasi-one-dimensional inviscid flow are in two categories. First, there are numerical limitations in obtaining solutions with the present code for different initial and boundary conditions. Second, the present coding limits what flow situations can be treated.

The numerical instability which occurs at low pressure ratios during the first few time steps can be severe. The assumption in the current code is that the shock forms immediately, which at high pressure ratios does not pose any problems. The mathematical modeling of the bursting diaphragm in subroutine "DBURST" adequately reduces the

transient instability for high pressure ratios, but not for low pressure ratios.

A second rather different numerical limitation was identified in numerically detecting the location of the head and tail of the expansion process. Because the expansion process is not a sharply defined front, fixing the precise locations of the head and tail waves numerically at a given time is difficult. However, the method currently used does accurately track the computed propagation of the wave along the characteristics. The characteristics are modeled currently as straight lines. A higher order curve would describe them more accurately.

The current code is limited to tracking two discontinuities and the expansion wave (i.e., a shock and a contact surface). Thus any situation which would generate another discontinuity can not be computed. The code can determine when this will occur, and will issue a message explaining the situation before terminating. Thus the shock colliding with a contact surface will currently terminate the program. A boundary pressure that is higher than the pressure inside the tube would also create a compression wave or possibly a shock wave. This condition will also terminate the program. Simulation of the process of a shock forming over an extended distance and time as compression waves pile on top of each other and strengthen, would also require additional code.

Finally, the variation of gamma that occurs across the contact surface in a wave rotor cannot be handled currently since E1DV2 assumes a single constant gamma throughout.

VI. CONCLUSIONS

Towards the development of a one-dimensional code for wave rotor applications, methods for tracking and correcting conditions across a contact discontinuity, applying open and closed-end boundary conditions, accounting for shock wave and contact surface interaction were devised and were presented here in detail. The EULER1 Fortran Code [Ref. 2] was revised to become the E1DV2 code with the following additional capabilities:

- 1) tracking of the contact surface and expansion wave
- 2) imposing high pressure initially on the right side of the diaphragm
- 3) correct jump conditions across the contact surface
- 4) allow open boundary conditions with constant pressure specified and allow exiting of shock and expansion waves
- 5) allow closed boundary conditions and model shock and expansion wave reflections
- 6) improved numerical accuracy from time zero to the maximum time step.

The code was tested on the shock tube problem under four different initial and boundary conditions with excellent results.

The following extensions are recommended to make the code suitable for wave rotor applications:

- 1) Solve the Riemann problem at the moment when the shock and contact surface intersect

- 2) Add additional code to track more than two discontinuities and one expansion wave
- 3) Incorporate a variable value of gamma into the code
- 4) Update the characteristic curve approximation from linear to a higher order polynomial
- 5) Add code necessary to handle open boundary conditions with inflow
- 6) Improve on the numerical computation at time zero for low pressure ratios across the diaphragm
- 7) Enable boundary conditions to be variable during program execution to allow wave rotor cycle design
- 8) Compare computations using the code with experimental data where available
- 9) Extend the code to two dimensions.

These extensions may require various degrees of effort. However, the ability of the QAZ1D solution method to be extended rather simply to describe viscous multi-dimensional flow suggests that such efforts would be justified.

APPENDIX A

DERIVATIONS OF EQUATIONS

A. LIST OF VARIABLES

| | |
|----------|---|
| A | Speed of sound |
| C_p | Specific heat at constant pressure |
| C_v | Specific heat at constant volume |
| e | specific internal energy |
| h | Specific enthalpy |
| P | Static pressure |
| Q | Modified Riemann variable |
| q_R | Reversible heat transferred |
| q | Velocity magnitude |
| R | Modified Riemann variable |
| R_G | Gas constant |
| S | Modified entropy (non-dimensional where $S = S \cdot R_G$) |
| T | Static temperature |
| t | Time |
| u | Velocity relative to a standing shock wave |
| v | Specific volume |
| W | Incoming Mach Number relative to a stationary shock wave |
| w | Work |
| ρ | Density |
| γ | Ratio of specific heats |

B. DERIVATION OF SHOCK JUMP EQUATION FOR HIGH PRESSURE ON THE RIGHT

The analytical equation relating the non-dimensionalized extended Riemann variable, R , change through a normal shock is derived below similar to that for the Q variable change when high pressure is on the left [Ref. 2:Appendix A].

Figure A.1 shows a shock moving to the left with velocity V_s . Subscript A is always associated with parameters to the left of any discontinuity, and B with those on the right. To enable normal shock relations to be used the system requires a velocity in the opposite direction but of equal magnitude be imposed upon it. The coordinate system was defined such that vectors, such as velocity, directed to the right are positive while those to the left are negative.

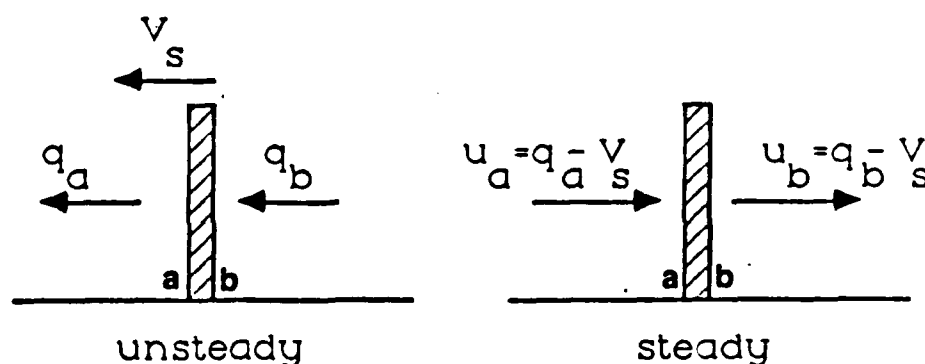


Figure A.1 Shock Wave with High Pressure on Right

Since relative incoming Mach Number, w , relates the velocity downstream, or that region into which the shock is traveling, to the speed of sound in that region [Ref. 10:pp. 114-154] then

$$w = \frac{q_A - V_S}{A_A} = \frac{u_A}{A_A} \quad (A1)$$

w is a positive value because q_A , though negative, is smaller in magnitude than the equally negative shock velocity, V_S . The speed of sound, A , is positive by definition.

The appropriate extended Riemann variable in this case is R , defined as

$$R = q - AS \quad (A2)$$

Since q is negative, this can be rewritten as

$$R = -(|q| + AS) \quad (A3)$$

to emphasize that R is the Riemann variable associated with the lesser change across the shock [Ref. 3:pp. 18-21]. We adopt the pattern of non-dimensionalizing velocity by the speed of sound, pressure and density by corresponding values downstream of the shock. Using the entropy S , defined by Verhoff [Ref. 1:p. 2] as

$$s = \frac{2}{\gamma-1} - \frac{1}{\gamma(\gamma-1)} \ln \left(\frac{P}{\rho^\gamma} \right) \quad (A4)$$

then

$$\begin{aligned} \frac{R_B - R_A}{A_A} &= \frac{q_B - A_B S_B}{A_A} - \frac{q_A - A_A S_A}{A_A} \\ &= \frac{q_B - q_A}{A_A} + S_A - \frac{A_B}{A_A} S_B \\ &= \frac{q_B - q_A}{A_A} + \left[\frac{2}{\gamma-1} - \frac{1}{\gamma(\gamma-1)} \ln \left(\frac{P_A}{P_A} \left(\frac{\rho_A}{\rho_A} \right)^\gamma \right) \right] \\ &\quad - \frac{A_B}{A_A} \left[\frac{2}{\gamma-1} - \frac{1}{\gamma(\gamma-1)} \ln \left(\frac{P_B}{P_A} \left(\frac{\rho_A}{\rho_B} \right)^\gamma \right) \right] \\ &= \frac{q_B - q_A}{A_A} + \left[1 - \frac{A_B}{A_A} \right] \left[\frac{2}{\gamma-1} \right] + \left(\frac{A_B}{A_A} \right) \frac{1}{\gamma(\gamma-1)} \left[\ln \left(\frac{P_B}{P_A} \left(\frac{\rho_B}{\rho_A} \right)^\gamma \right) \right] \end{aligned} \quad (A5)$$

The ratios of pressure, density, and sonic velocity across a normal shock from Zucker [Ref. 7:p. 151] and Shapiro [Ref. 10:p. 118] in terms of Mach Number are

$$\frac{P_B}{P_A} = \frac{2\gamma}{\gamma+1} w^2 - \frac{\gamma-1}{\gamma+1} \quad (A6)$$

$$\frac{\rho_B}{\rho_A} = \frac{(\gamma+1)w^2}{(\gamma-1)w^2 + 2} \quad (A7)$$

$$\frac{A_B}{A_A} = \frac{1}{(\gamma+1)w} \left[2(\gamma-1) \left[1 + \frac{\gamma-1}{2} w^2 \right] \left[\frac{2\gamma}{\gamma-1} w^2 - 1 \right] \right]^{1/2} \quad (A8)$$

The first term on the right side of equation (A5) can be expressed in terms of w by applying simple continuity in mass.

$$\rho_A u_A \text{Area}_A = \rho_B u_B \text{Area}_B \quad (\text{A9})$$

Taking the areas as equal, this becomes

$$\frac{\rho_A}{\rho_B} = \frac{u_B}{u_A} = \frac{(\gamma-1)w^2+2}{(\gamma+1)w^2} \quad (\text{A10})$$

By subtracting one from each side

$$\begin{aligned} \frac{u_B}{u_A} - 1 &= \frac{\gamma-1}{\gamma+1} \frac{w^2+2}{w^2} - 1 \\ \frac{u_B - u_A}{u_A} &= \frac{(\gamma-1)w^2+2 - (\gamma+1)w^2}{(\gamma+1)w^2} \\ \frac{u_B - u_A}{u_A} &= \frac{2(1-w^2)}{(\gamma+1)w^2} \end{aligned} \quad (\text{A11})$$

Substitute equation (A1) for u_A to get

$$\begin{aligned} \frac{u_B - u_A}{w_A} &= \frac{2(1-w^2)}{(\gamma+1)w^2} \\ \frac{u_B - u_A}{A_A} &= \frac{2(1-w^2)}{(\gamma+1)w} \end{aligned} \quad (\text{A12})$$

thus

$$\begin{aligned} \frac{q_B - V_s - (q_A - V_s)}{A_A} &= \frac{2(1-w^2)}{(\gamma+1)w} \\ \frac{q_B - q_A}{A_A} &= \frac{2(1-w^2)}{(\gamma+1)w} \end{aligned} \quad (\text{A13})$$

Combining equations (A6), (A7), (A8), and (A13) into equation (A5) gives

$$\begin{aligned} \frac{R_B - R_A}{A_A} = & \frac{2(1-w^2)}{(\gamma+1)w} + \left(\frac{2}{\gamma-1}\right) \left\{ 1 - \left(\frac{1}{(\gamma+1)w}\right) \left(2(\gamma-1) \left[1 + \frac{\gamma-1}{2} w^2 \right] \left[\frac{2\gamma}{\gamma-1} w^2 - 1 \right] \right)^{1/2} \right. \\ & + \left. \left(\frac{1}{(\gamma+1)w}\right) \left(2(\gamma-1) \left[1 + \frac{\gamma-1}{2} w^2 \right] \left[\frac{2\gamma}{\gamma-1} w^2 - 1 \right] \right)^{1/2} \right\} \\ & \times \left\{ \frac{1}{\gamma(\gamma-1)} \left[\ln \left(\left(\frac{2\gamma}{\gamma+1} w^2 - \frac{\gamma-1}{\gamma+1} \right) \left(\frac{(\gamma-1)w^2 + 2}{(\gamma+1)w^2} \right) \right) \right] \right\} \end{aligned} \quad A.14$$

C. DERIVATION OF CONTACT SURFACE JUMP EQUATION

The analytical expression for the ratio of son. velocity across a constant surface is derived, as follows.

The first law of thermodynamics is stated as

$$\left[\begin{array}{c} \text{change of} \\ \text{internal energy} \end{array} \right] = \left[\begin{array}{c} \text{incremental} \\ \text{amount of} \\ \text{heat added} \\ \text{to system} \end{array} \right] + \left[\begin{array}{c} \text{work done} \\ \text{on the} \\ \text{system by} \\ \text{surrounding} \end{array} \right]$$

or, in differential form

$$de = \partial q_R + \partial W \quad (A15)$$

Internal energy is defined as

$$e = h - pv \quad (A16)$$

then

$$de = dh - pdv - vdp \quad (A17)$$

The incremental work is given by

$$\partial W = pdv \quad (A18)$$

The heat addition is determined from the definition of modified entropy,

$$d\bar{S} = -\frac{1}{\gamma} \frac{dq_R}{T}$$

$$dq_R = -\gamma T d\bar{S} \quad (A19)$$

Substituting equations (A17), (A18), and (A19) into equation (A15) yields

$$dh - pdv - vdp = -\gamma T d\bar{S} - pdv$$

Canceling like terms, and rearranging gives

$$dp = \frac{\gamma}{v} T d\bar{S} + \left(\frac{1}{v}\right) dh \quad (A20)$$

Enthalpy, h , is defined as

$$h = C_p T \quad (A21)$$

For a perfect gas the sonic velocity is given by

$$A = (\gamma R_G T)^{1/2}$$

thus

$$dA = \frac{1}{2} (\gamma R_G T)^{1/2} (dT/T) \quad (A22)$$

Substituting equation (A21) and then (A22) into equation (A20), equation (A20) becomes

$$\begin{aligned} dp &= \frac{\gamma}{v} T d\bar{S} + \left(\frac{1}{v}\right) d(C_p T) \\ &= \frac{\gamma}{v} T d\bar{S} + \left(\frac{1}{v}\right) C_p dT \\ &= \frac{\gamma}{v} T d\bar{S} + \left(\frac{1}{v}\right) C_p T \left(\frac{2dA}{A}\right) \end{aligned} \quad (A23)$$

For an ideal gas,

$$\gamma = C_p / C_v$$

and

$$C_p = C_v + R_G$$

Combining these two equations, and rearranging

$$C_p = R_G \left(\frac{\gamma}{\gamma-1} \right) \quad (A24)$$

Substituting equation (A24) into equation (A23), and observing that pressure remains constant through the contact surface

$$\frac{T}{v} \gamma d\bar{S} + \frac{T}{v} \left(R_G \left(\frac{2\gamma}{\gamma-1} \right) \right) \frac{dA}{A} = 0 \quad (A25)$$

Eliminating T/v , equation (A25) becomes

$$\left(\frac{2}{\gamma-1} \right) d(\bar{S}/R_G) = - \frac{dA}{A} \quad (A26)$$

Then \bar{S}/R_G becomes a non-dimensionalized entropy. Using the notation of $S = \bar{S}/R_G$ now for the non-dimensionalized form, so that by integrating both sides

$$\frac{2}{\gamma-1} \int_A^B dS = \int_A^B - \frac{dA}{A}$$

$$\frac{2}{\gamma-1} [S_B - S_A] = \ln A_A/A_B$$

which can be written as

$$A_A/A_B = \exp \left(\frac{(\gamma-1)}{2} (S_B - S_A) \right) \quad (A27)$$

Definitions used in this development, excluding modified entropy were from [Ref. 11:pp. 87-125].

D. DERIVATION OF EXTENDED RIEMANN VARIABLE CHANGE ACROSS A CONTACT SURFACE

An analytical expression for the change in the non-dimensionalized extended Riemann variable, Q , across a contact surface traveling right is derived here. Figure A.2 depicts a contact surface moving right with velocity q .

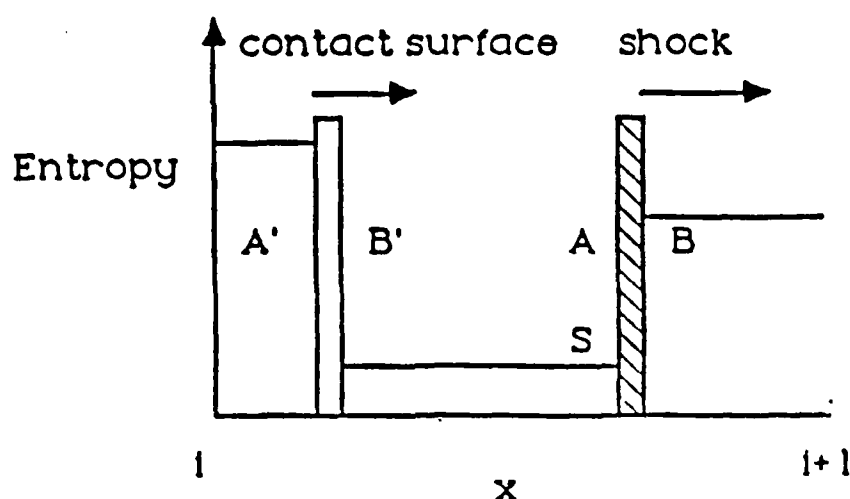


Figure A.2 Contact Surface Traveling Right
Subscript Notation

Values of parameters to the left of the interface are denoted by subscript A' , and those to the right with subscript B' . As illustrated, parameters to the left of the shock have subscript A , and to the right subscript B . All velocities are non-dimensionalized by sonic velocity immediately downstream of the discontinuity. Thus using the definition of the extended Riemann variable,

$$Q = q + AS \quad (A28)$$

then

$$\begin{aligned} \frac{Q_{A'} - Q_{B'}}{A_{B'}} &= \frac{q_{A'} + A_{A'} S_{A'}}{A_{B'}} - \frac{q_{B'} + A_{B'} S_{B'}}{A_{B'}} \\ &= \frac{q_{A'} - q_{B'}}{A_{B'}} + \frac{A_{A'}}{A_{B'}} (S_{A'}) - S_{B'} \end{aligned} \quad (A29)$$

Since velocity is constant through a contact surface, so that

$$q_{A'} = q_{B'}$$

then equation (A29) becomes

$$\frac{Q_{A'} - Q_{B'}}{A_{B'}} = \frac{A_{A'}}{A_{B'}} S_{A'} - S_{B'}$$

Using equation (A27), this can be written

$$\frac{Q_{A'} - Q_{B'}}{A_{B'}} = \exp^{\left(\frac{(\gamma-1)}{2} (S_{B'} - S_{A'})\right)} S_{A'} - S_{B'} \quad (A30)$$

Multiply each side by $A_{B'}/A_{B'}$, and since $A_{B'} = A_A$ then

$$\frac{Q_{A'} - Q_{B'}}{A_B} = [(\exp^{\left(\frac{(\gamma-1)}{2} (S_{B'} - S_{A'})\right)} (S_{A'}) - S_{B'})] (A_A/A_B) \quad (A31)$$

For a contact surface traveling to the left, as illustrated in Fig. A.3, the derivation is entirely similar but

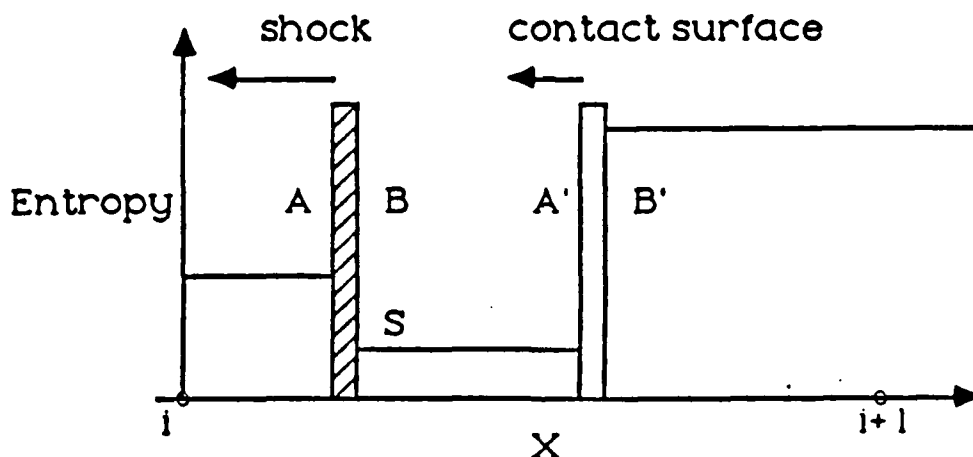


Figure A.3 Contact Surface Traveling Left
Subscript Notation

using the R, extended Riemann variable with the result

$$\frac{R_{B'} - R_{A'}}{A_A} = \left[\exp\left(\frac{(\gamma-1)}{2} (S_{A'} - S_{B'})\right) (S_{B'} - S_{A'}) \right] \left(\frac{A_{B'}}{A_A}\right) \quad (A32)$$

E. DERIVATION OF OPEN BOUNDARY CONDITIONS WITH CONSTANT PRESSURE

The equation for density and temperature ratios in terms of pressure and entropy are derived below. The definition of modified entropy in non-dimensional form is

$$S = \frac{2}{\gamma-1} - \frac{1}{\gamma(\gamma-1)} \ln\left(\frac{P}{\rho^\gamma}\right) \quad (A33)$$

where P and ρ are non-dimensionalized ratios. Then

$$\left(S - \frac{2}{\gamma-1}\right)(-\gamma(\gamma-1)) = \ln(P/\rho^\gamma)$$

or

$$\exp\left[\left(S - \frac{2}{\gamma-1}\right)(-\gamma(\gamma-1))\right] = P/\rho^\gamma$$

Rearranging,

$$\rho^\gamma = [P \exp\left[\left(S - \frac{2}{\gamma-1}\right)(-\gamma(\gamma-1))\right]]$$

thus

$$\rho = P^{1/\gamma} \left(\exp\left[\left(S - \frac{2}{\gamma-1}\right)(-\gamma(\gamma-1))\right] \right)^{-1/\gamma}$$

$$\rho = \left\{ \frac{1}{P} \left(\exp\left[\left(S - \frac{2}{\gamma-1}\right)(-\gamma(\gamma-1))\right] \right) \right\}^{-1/\gamma} \quad (A34)$$

Using the perfect gas law,

$$\rho = \left\{ \frac{1}{\rho T} \left(\exp\left[\left(S - \frac{2}{\gamma-1}\right)(-\gamma(\gamma-1))\right] \right) \right\}^{-1/\gamma}$$

$$\rho^{-\gamma} = \frac{1}{\rho T} \left(\exp\left[\left(S - \frac{2}{\gamma-1}\right)(-\gamma(\gamma-1))\right] \right)$$

multiply each side by T and divide by $\rho^{-\gamma}$ then

$$T = \rho^{\gamma}/\rho(\exp^{(S - \frac{2}{\gamma-1})(-\gamma(\gamma-1))})$$

thus

$$T = \rho^{\gamma-1}[\exp^{(S - \frac{2}{\gamma-1})(\gamma(1-\gamma))}] \quad (A35)$$

APPENDIX B

OPERATION OF E1DV2 ON THE NPS VM/CMS SYSTEM

A. TERMINAL REQUIREMENTS

Terminal requirements depend on the desired output. Any terminal, such as the IBM 3278, connected into the VM/CMS system is adequate for a tabular listing or Disspla Metafile of data. The Disspla Metafile stores graphical data for display using DISSPOP commands. If output of the plots on a monitor screen is desired, an IBM 3277-TEK618 dual screen terminal must be used. Table B.I lists terminal requirements for different outputs.

TABLE B.I

TERMINAL AND OUTPUT

| <u>OUTPUT</u> | <u>TERMINAL</u> |
|--|-----------------|
| Tabular listing of data | Any terminal |
| Graphical data in format for VERSATEC printing | Any terminal |
| Graphical data plotted on screen | IBM 3277-TEK618 |

B. PROCEDURE

1. Editing Variables and Program Setup

To change the initial and boundary conditions, graphical output, and grid size to run the program under

different conditions, the following must be done. First, pick the appropriate terminal from Table B.I and log on. Enter the editor by issuing the command

x E1DV2 FORTRAN A

Table B.II lists the code variables that will require editing and their meaning. In addition, to these variables if graphical plots are to be outputted then enter the "BORDER" and "EXACT" subroutines and edit the lines:

```
"FIRST ORDER  N = ???"  
"DENSITY RATIO = ???  TEMP RATIO = ???"  
"PRESSURE RATIO = ???"
```

To issue the output graphs to the screen comment out the lines "COMPRS", and use

CALL TEK618

Otherwise, if a Disspla Metafile of the graphics plot is desired, comment out the "TEK618" line, and use

CALL COMPRS

These lines are in the "Set up graphics plot of variable" loop in the main program.

To store these changes, hit the enter key, and type "file". Select the enter key once more. The program is now ready to be compiled and executed.

2. Commands

To compile and execute the E1DV2 code on the VM/CMS system perform the following commands exactly as typed:

TABLE B.II
EDITABLE VARIABLES

| <u>Variable</u> | <u>Edit</u> |
|--|--|
| DIMENSION statement for arrays A(N),..., XARRAY(N) | set the dimension of the arrays equal to the number of nodes in the grid |
| N | set to the number of nodes in the grid |
| GRAPHS | set to: 0 for tabular listing of data 1 for plot of density, entropy, pressure and velocity distributions 2 for plot of exact density distribution compared to computed density distribution |
| SKIP | set to number of time steps between calls to output routines |
| JSTOP | set to maximum number of time steps |
| TRI | set to initial temperature ratio across diaphragm |
| PRI | set to initial pressure ratio across diaphragm |
| DRI | set to initial density ratio across diaphragm |
| QLI | set to initial velocity left of diaphragm |
| QRI | set to initial velocity right of diaphragm |
| LBDPRI | set to initial pressure ratio across left boundary |
| LBDTRI | set to initial temperature ratio across left boundary |
| LBDORI | set to initial density ratio across left boundary |

TABLE B.II (CONTINUED)

| | |
|------------------|--|
| RBDDRI | set to initial density ratio across right boundary |
| RBDPRI | set to initial pressure ratio across right boundary |
| RBDTRI | set to initial temperature ratio across right boundary |
| G | set to desired value of gamma |
| EE | set to desired error tolerance for calculation of characteristic slope (i.e., $0.1D^{-8}$) |
| LWPRES | set to: 2 for low pressure on right side of diaphragm 3 for low pressure on left side |
| LBNDRY RBNDRY | set to: 1 for closed boundary 0 for open boundary |
| LBDPRS RBDPRS | set to: 0 for constant pressure at left boundary 1 for pressure that adjusts at the left boundary |
| XINIT | set to location of diaphragm |
| VHEAD | set to exact velocity for head of expansion wave |
| VTAIL | set to exact velocity for tail of expansion wave |
| VCDE | set to exact velocity for contact surface |
| VSE | set to exact velocity for shock wave |
| DLCD | set to exact density behind contact surface |
| DLSH | set to exact density behind shock |

TABLE B.II (CONTINUED)

SIGMA(1,2)
 SIGMA(2,2)
 SIGMA(3,2)
 SIGMA(4,2)

set to diaphragm location (i.e.,
 0.5D00)

Y

There are four cases where Y
 appears in the program
 edit as follows:
 Comment out Y = (Integer#), use:
 first, $Y = (N+1)/2$ if LWPRES = 2
 second, $Y = (N+3)/2$
 and
 third, $Y = (N-1)/2$ if LWPRES = 3
 fourth, $Y = (N+1)/2$
 for diaphragm at 0.5
 Comment out those above, if dia-
 phragm at another node. Set
 first, Y = (Integer #) of node for
 LWPRES = 2
 second, $Y = (\# + 1)$
 and
 third, $Y = (\text{Integer } \# - 1)$ of node
 for LWPRES = 3
 fourth, $Y = (\#)$

- 1) Increase the virtual memory by entering

DEFINE STORAGE 1M

- 2) To return to CMS environment enter

I CMS

- 3) To compile the program enter

FORTVS E1DV2

The screen will display messages as it compiles each routine and when finished a ready symbol appears.

- 4) To execute the program, enter

DISSPLA E1DV2

The message

...YOUR FORTRAN PROGRAM IS NOW BEING LOADED...
...EXECUTION WILL SOON FOLLOW...

should appear, followed by

...EXECUTION BEGINS...

If at a TEK618 terminal with GRAPHS equal to 1 or 2 then the screen on the TEK618 will begin plotting the selected graph. A

...press ENTER to continue...

message will appear on the 3277 terminal. If a copy of the plot is desired, do so now before pressing the enter key. After pressing the enter key on the 3277 terminal, the plot will be erased and the program will terminate. Proper termination will result in

END OF DISSPLAY 9.2 #### VECTORS IN 1 PLOT...

appearing, followed by a ready message.

If GRAPHS was set to 0, then proper termination would be a ready message. The tabular listing of pressure, density, velocity, entropy, and Riemann variables will be in "FILE FT09F001." The exact location of the shock, contact surface, and expansion wave with elapsed time will be in "FILE FT08F001." The computed location of the shock, contact surface, and expansion wave with elapsed time will be in "FILE FT10F001."

A second method to compile and execute the program, plus provide the files with a name is to create the following EXEC file on the user's disk.

```
FI 9 DISK FILE09 LISTING A(PERM
FI 10 DISK FILE10 LISTING A(PERM
FI 8 DISK FILE08 LISTING A(PERM
FORTVS &1
```

A possible file name and required file type would be

RUN EXEC

To compile the program and define the output files, enter

RUN E1DV2

After compiling is finished, and the ready message appears, enter

DISSPLA E1DV2

The program executes as outlined before.

APPENDIX C

FLOWCHARTS

Flowcharts for major routines in E1DV2 are shown.

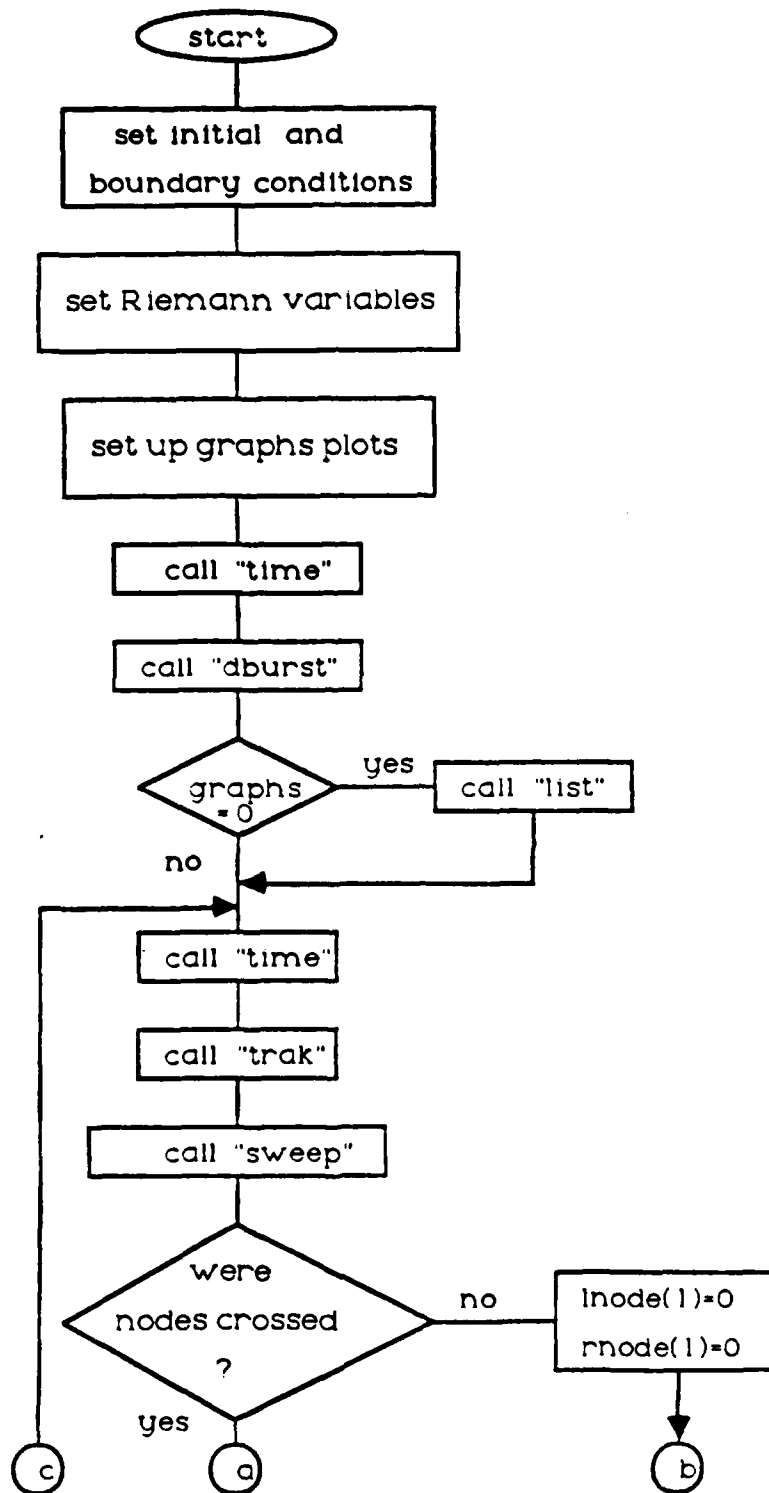


Figure C.1 Main Program Flowchart

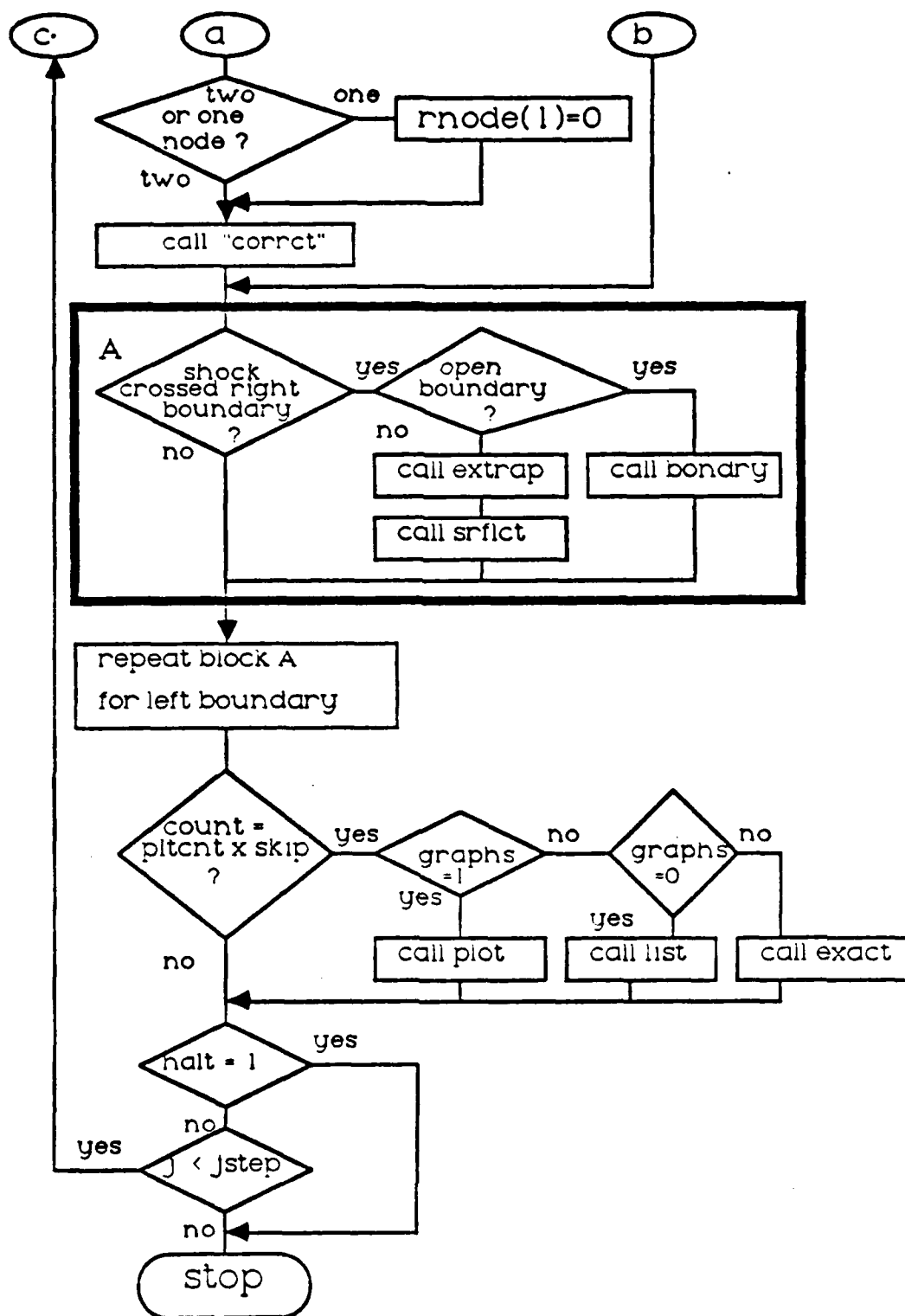


Figure C.1 (Continued)

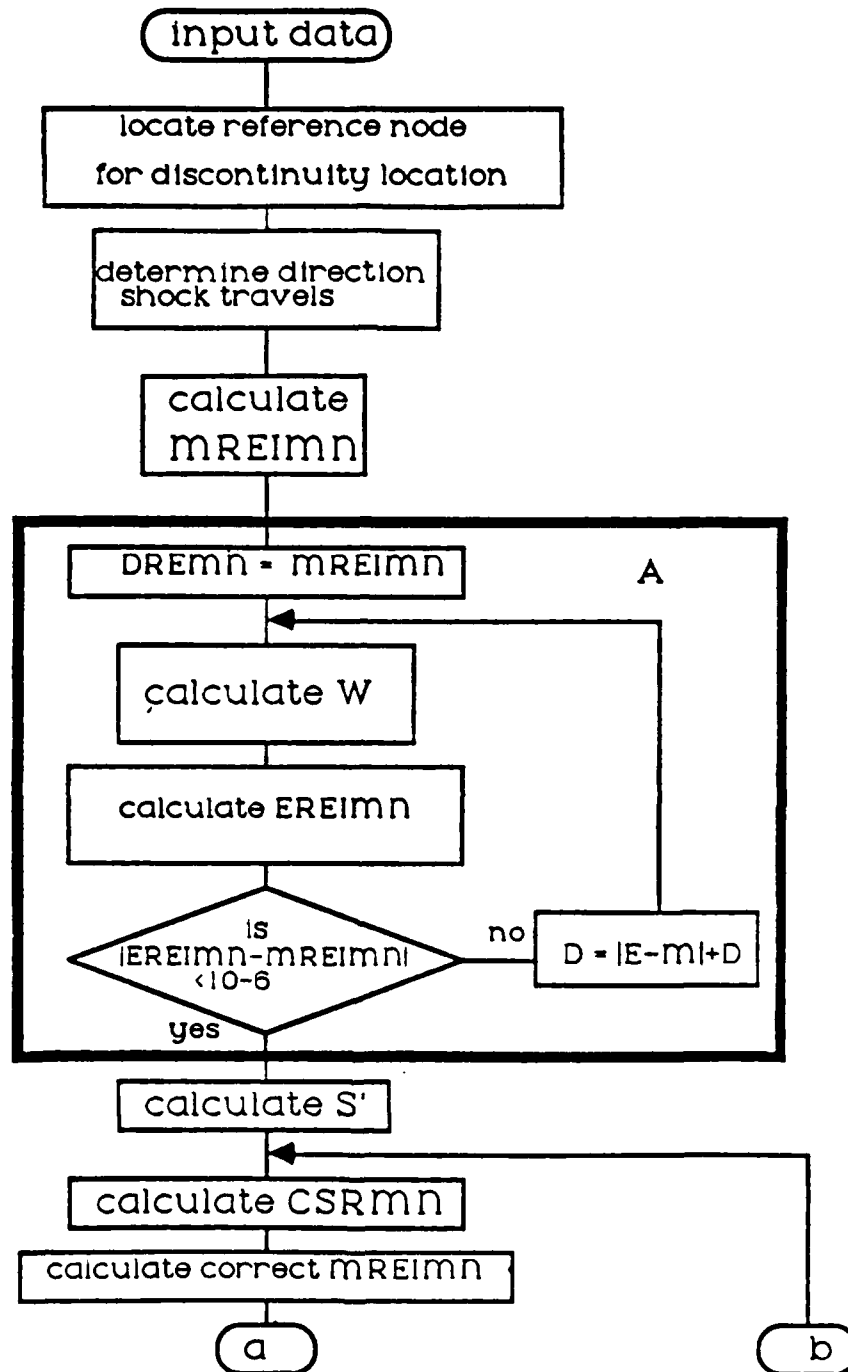


Figure C.2 "DBURST" Subroutine Flowchart.

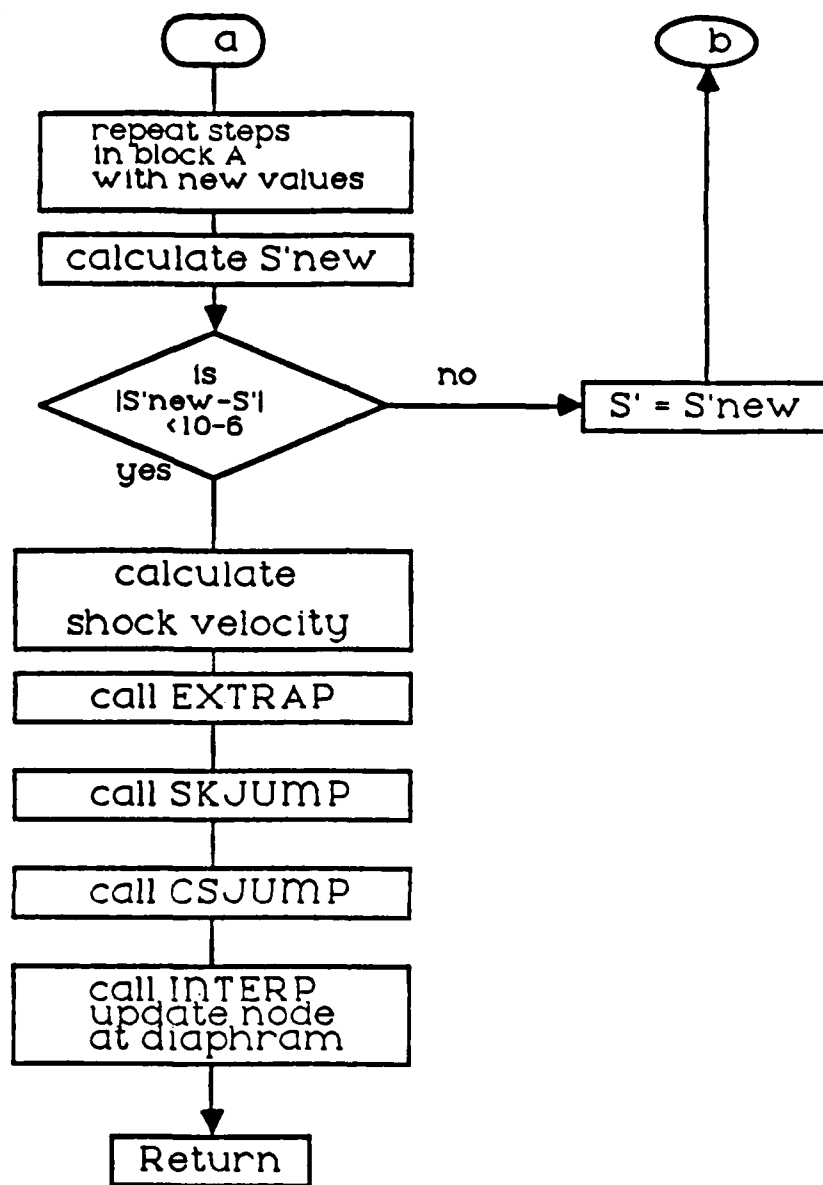


Figure C.2 (Continued)

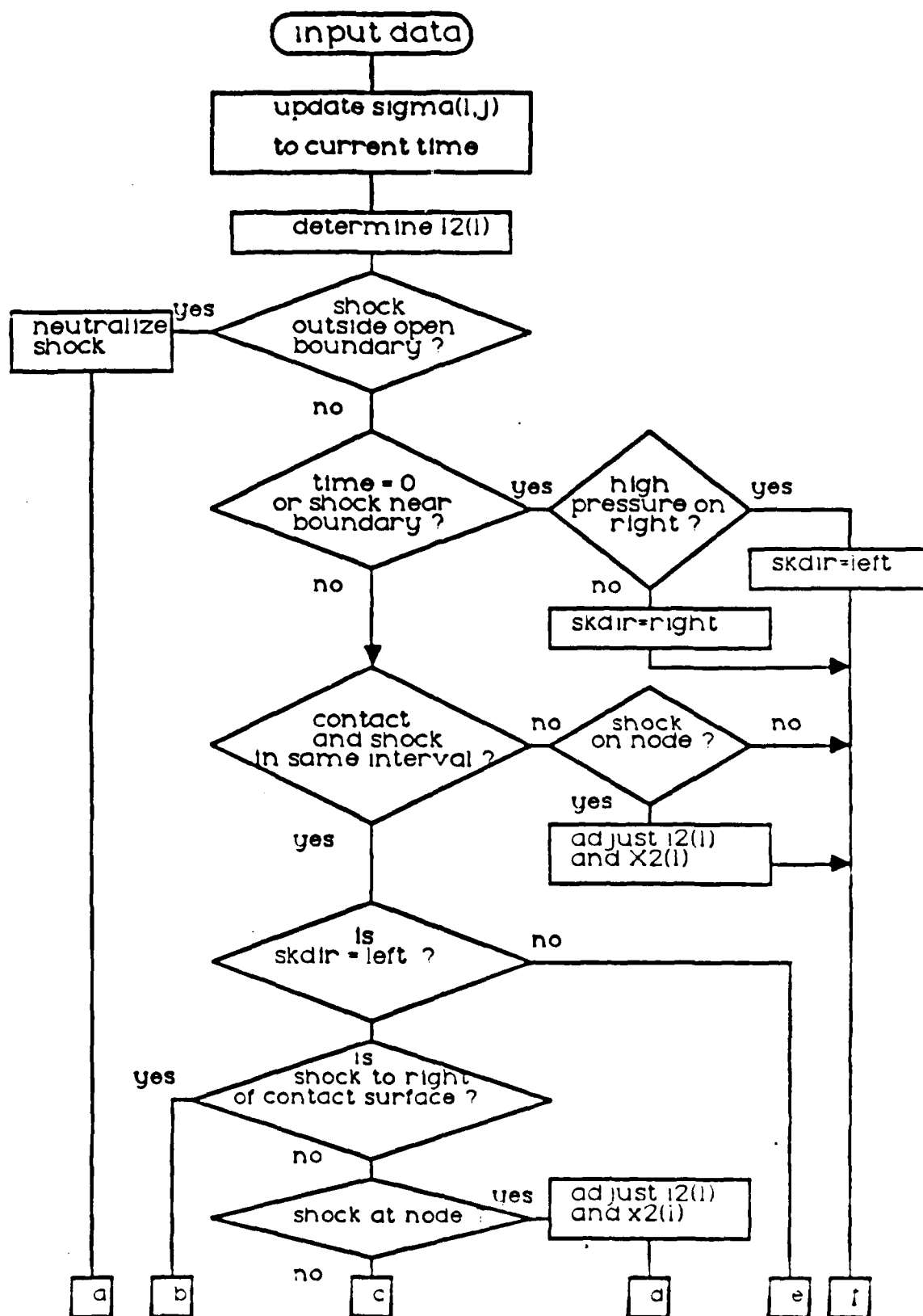


Figure C.3 "TRAK" Subroutine Flowchart

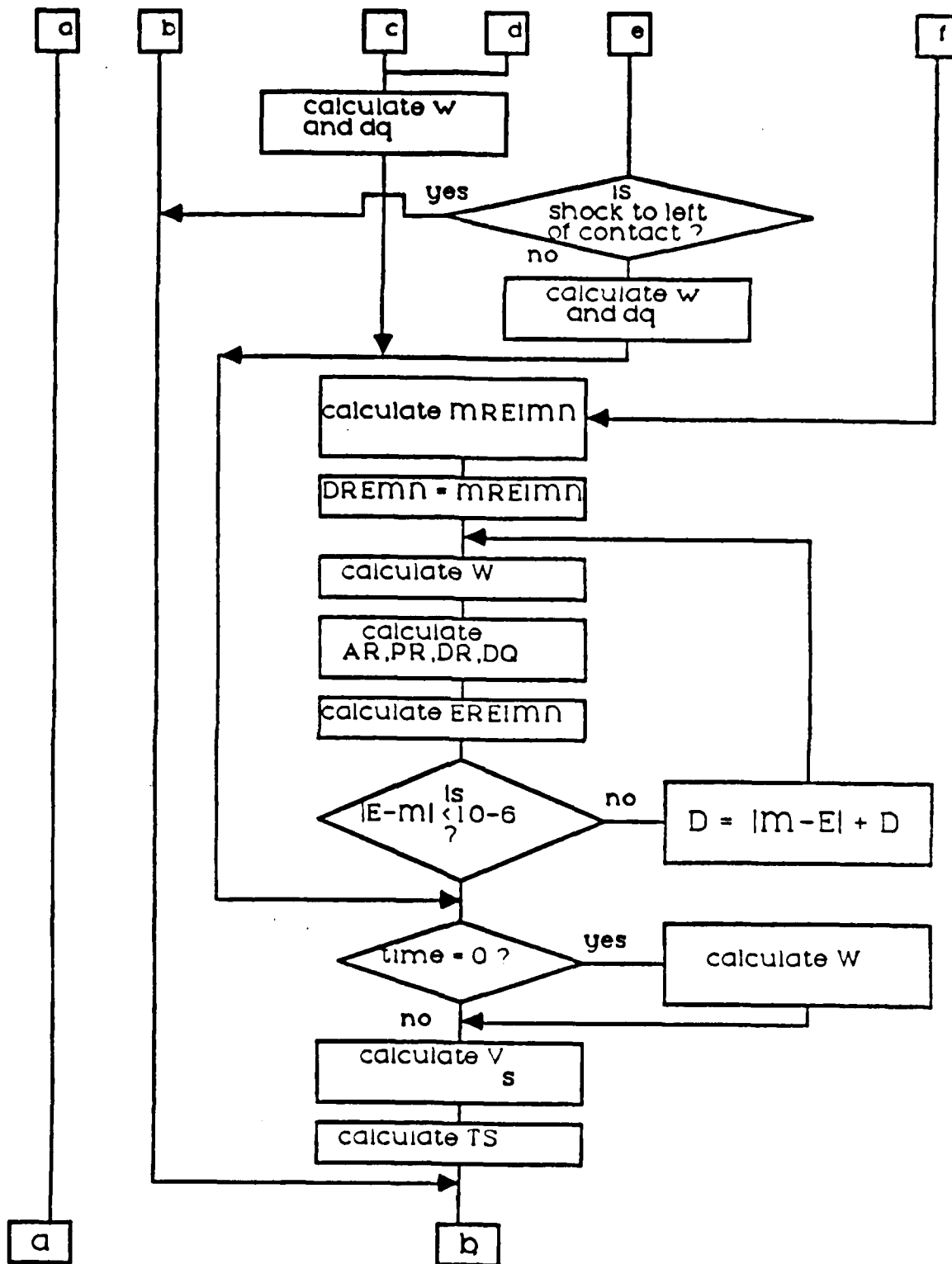


Figure C.3 (Continued)

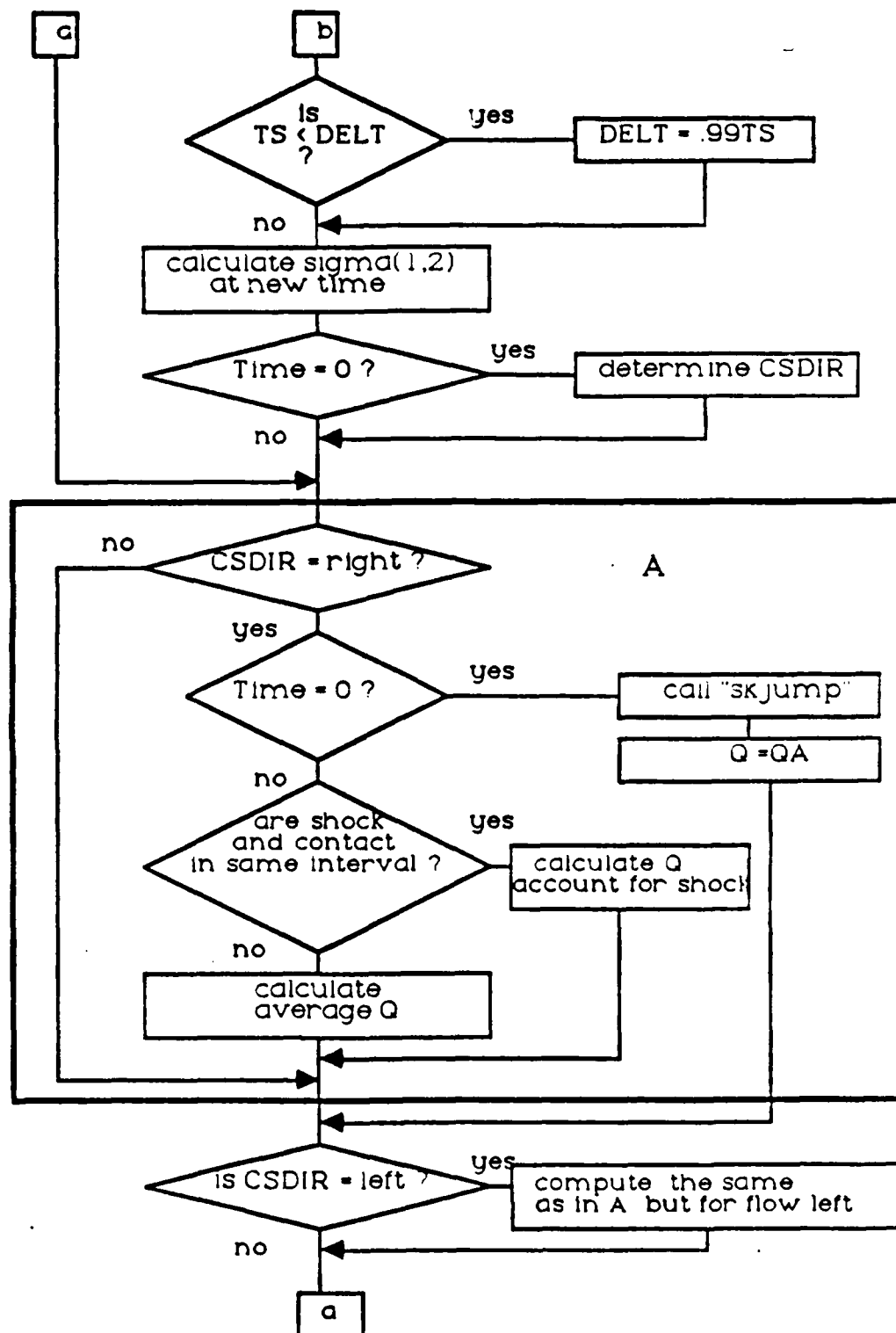


Figure C.3 (Continued)

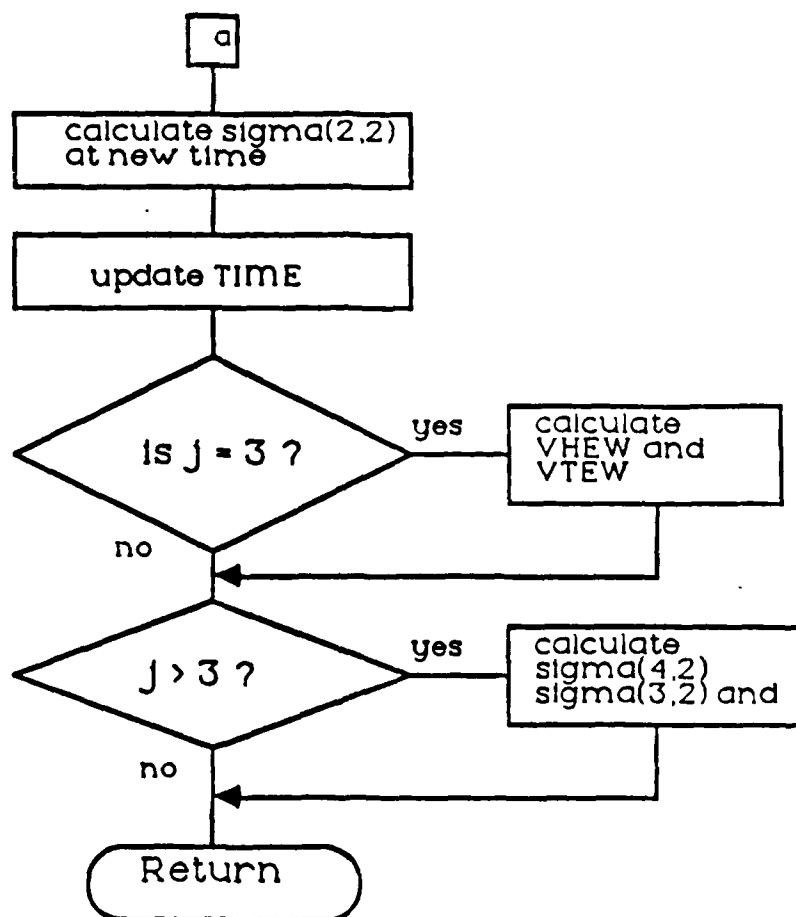


Figure C.3 (Continued)

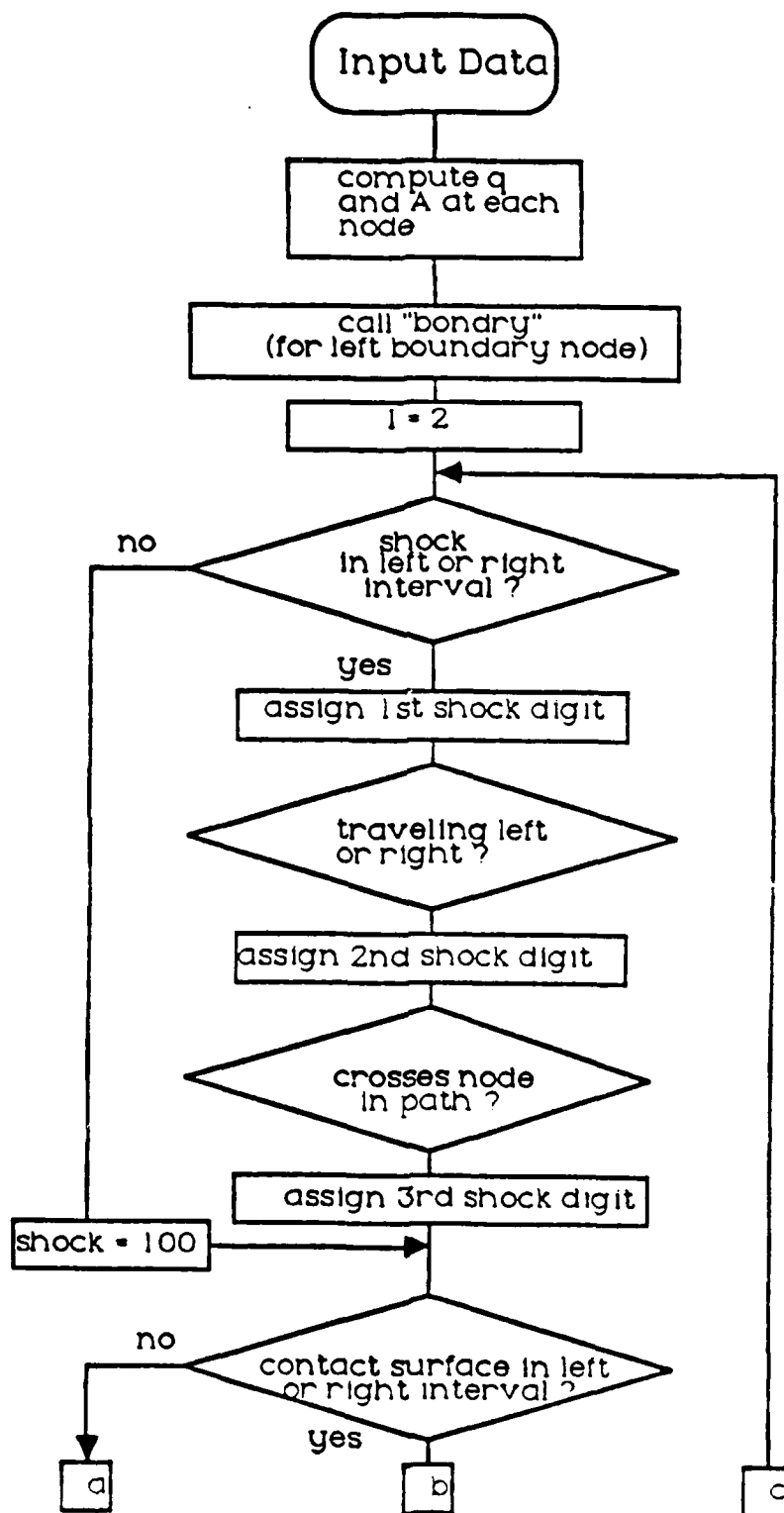


Figure C.4 "SWEEP" Subroutine Flowchart

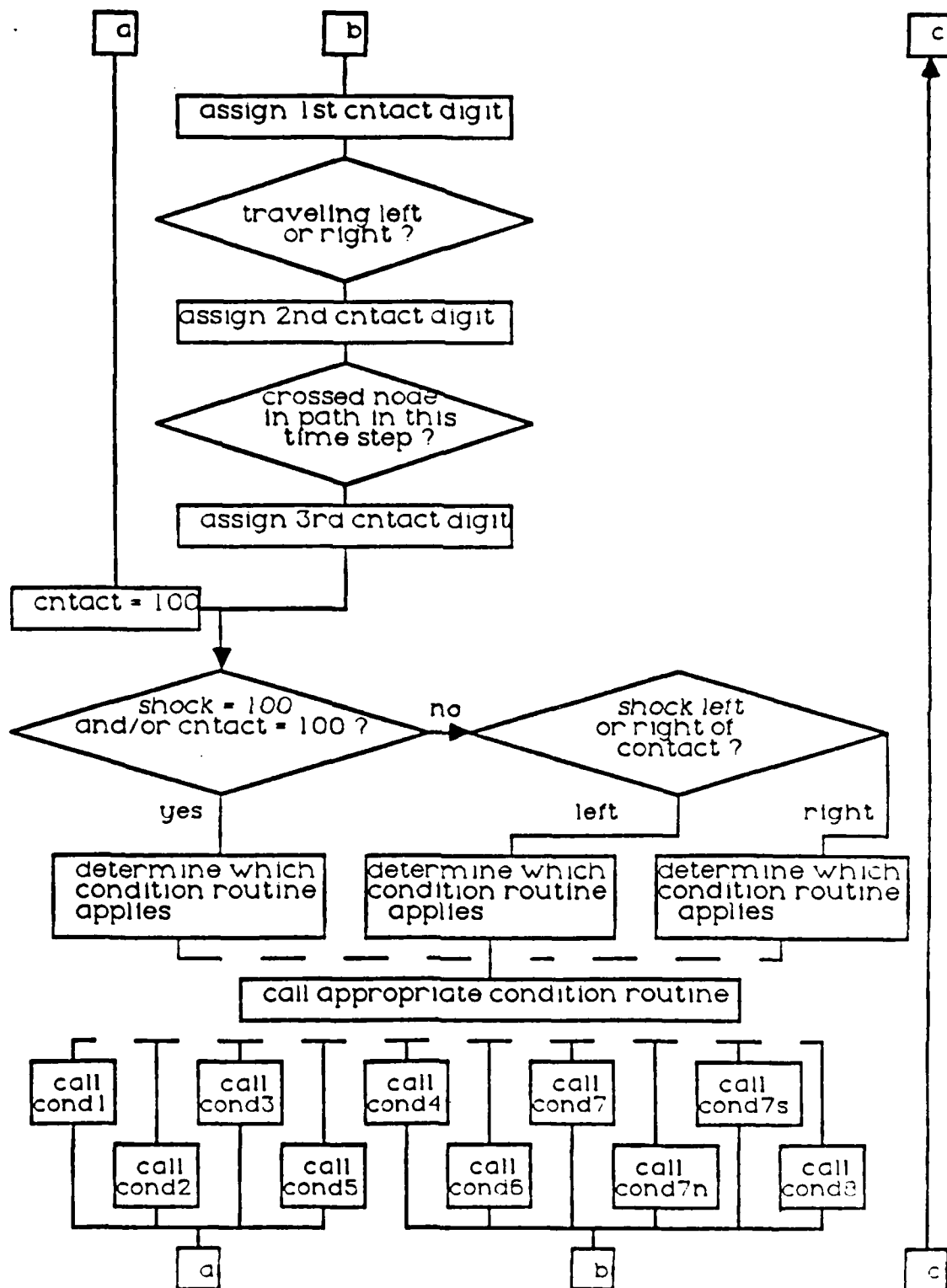


Figure C.4 (Continued)

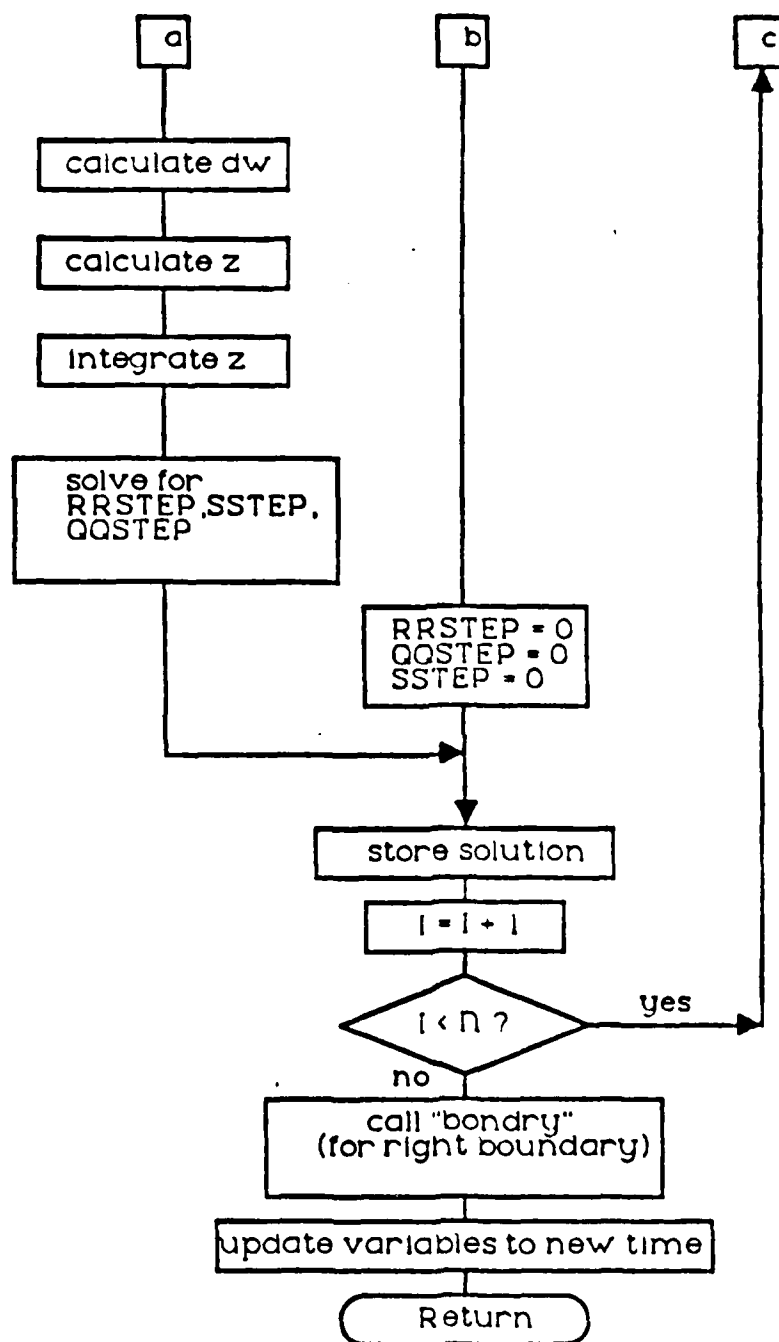


Figure C.4 (Continued)

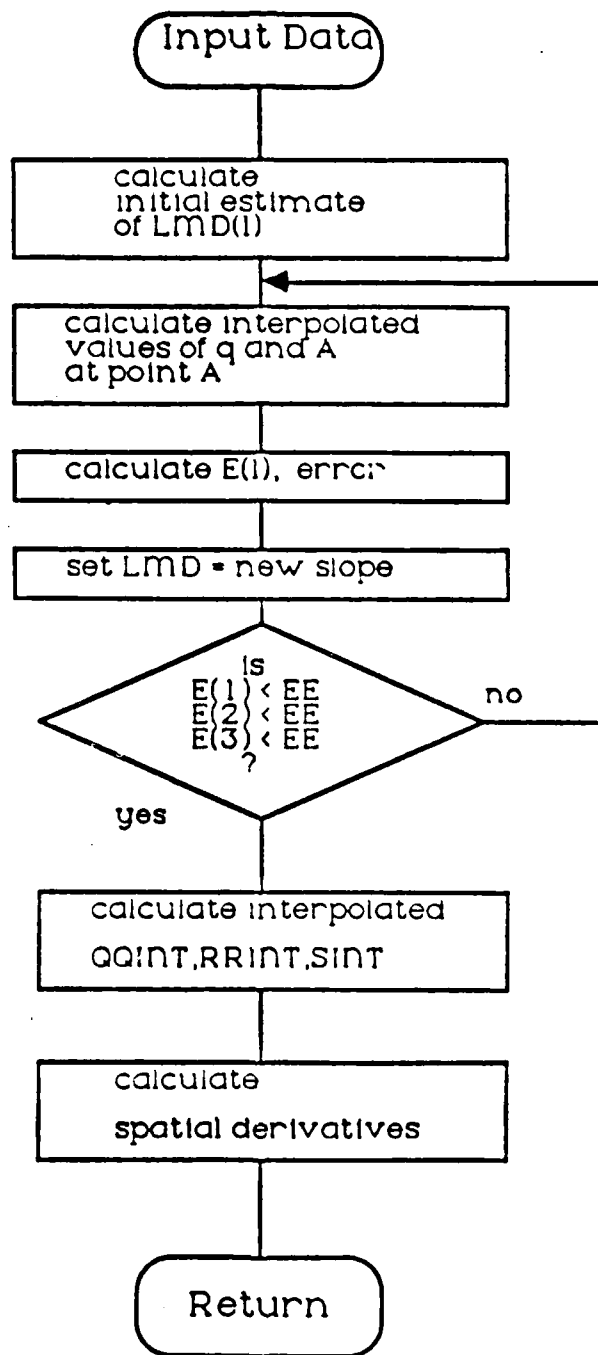


Figure C.5 General "COND1, 2, 3, and 5" Subroutine Flowchart

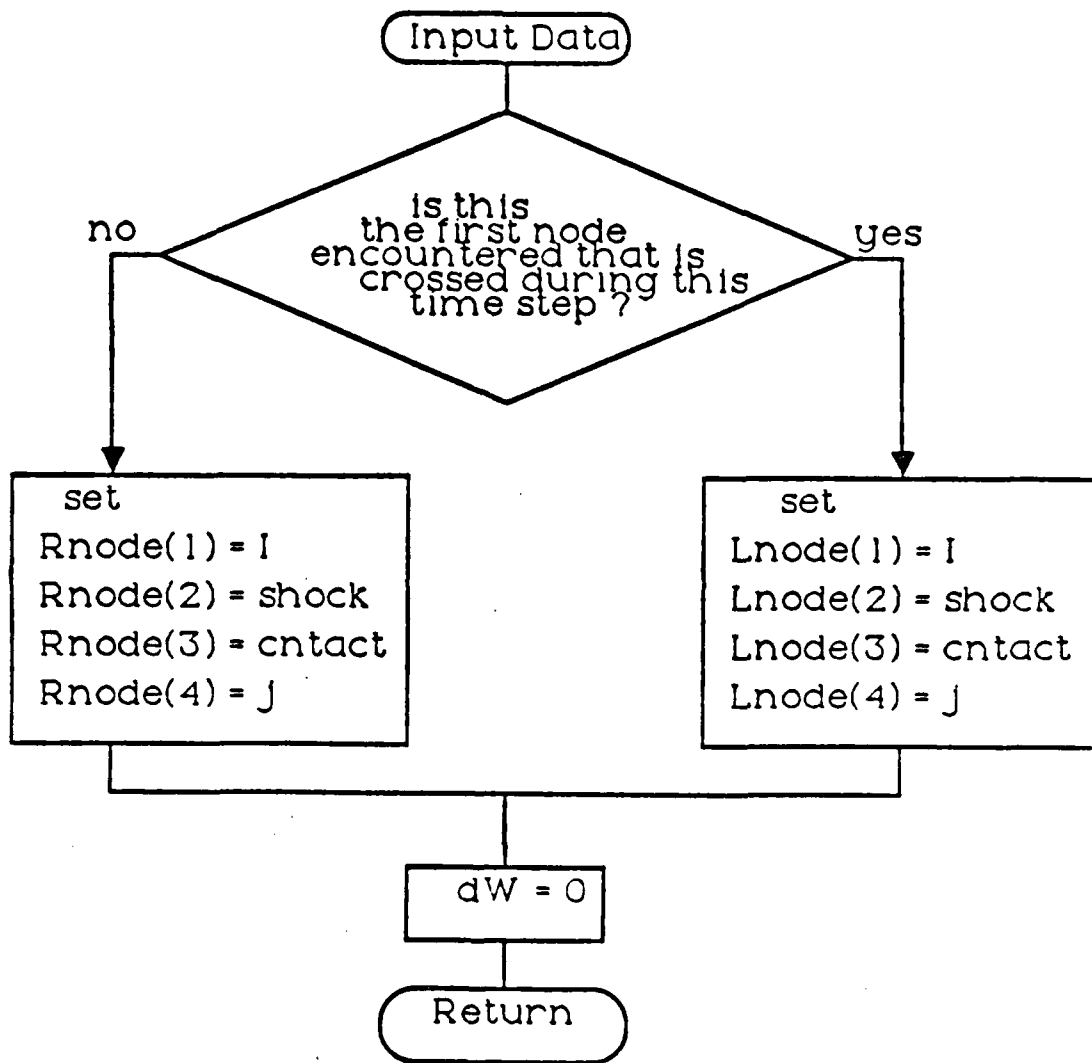


Figure C.6 "COND4" Subroutine Flowchart

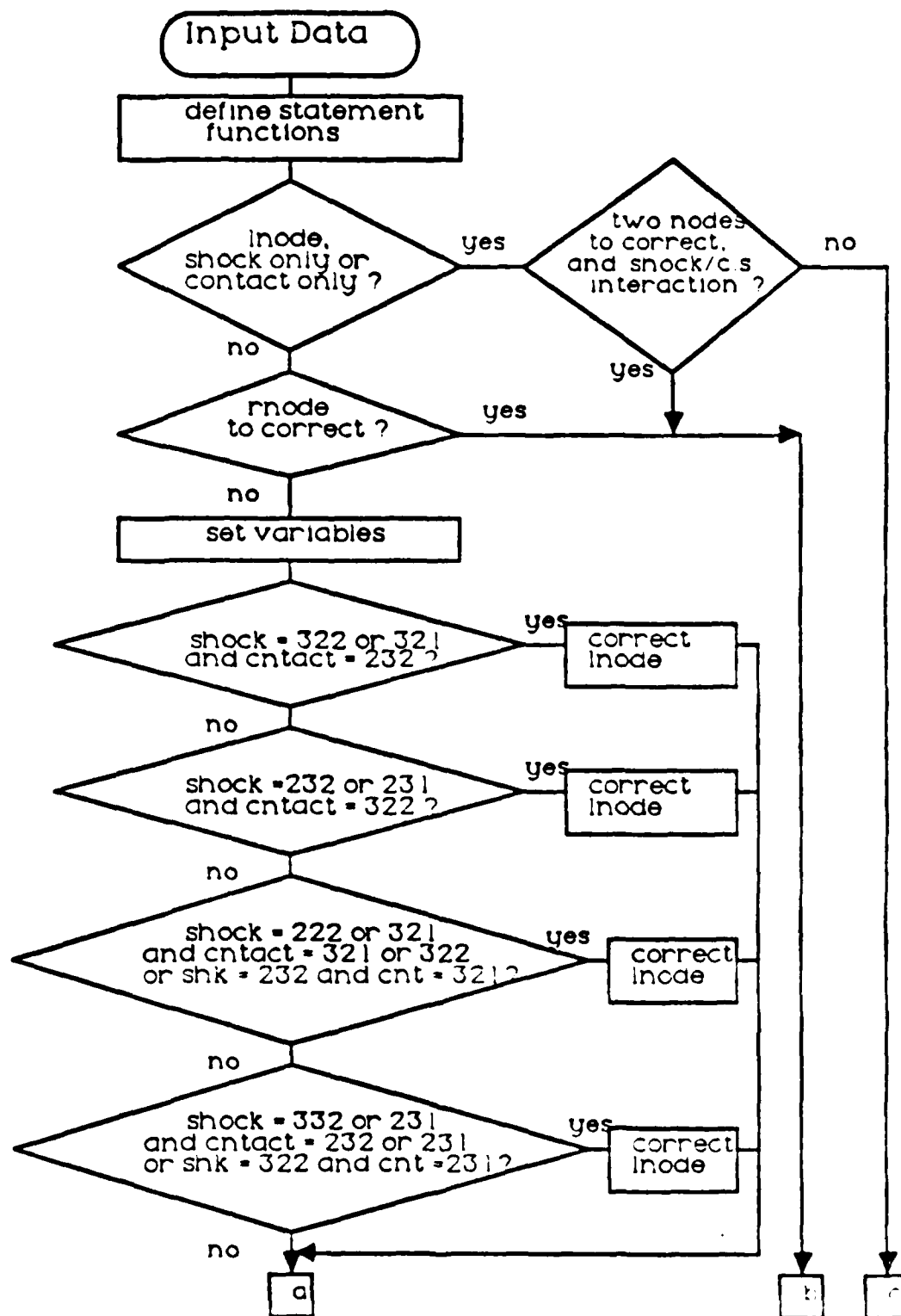


Figure C.7 "CORRECT" Subroutine Flowchart

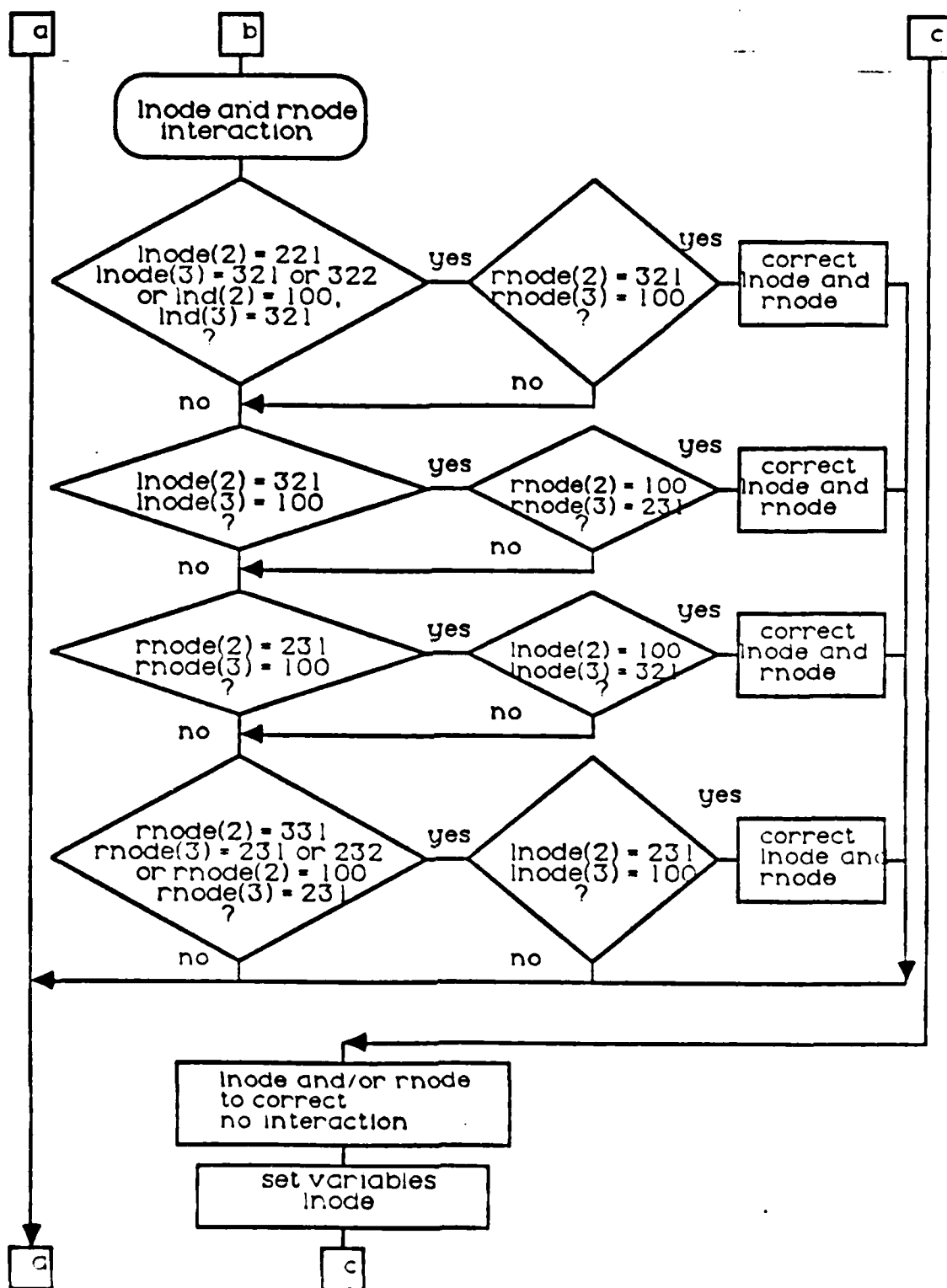


Figure C.7 (Continued)

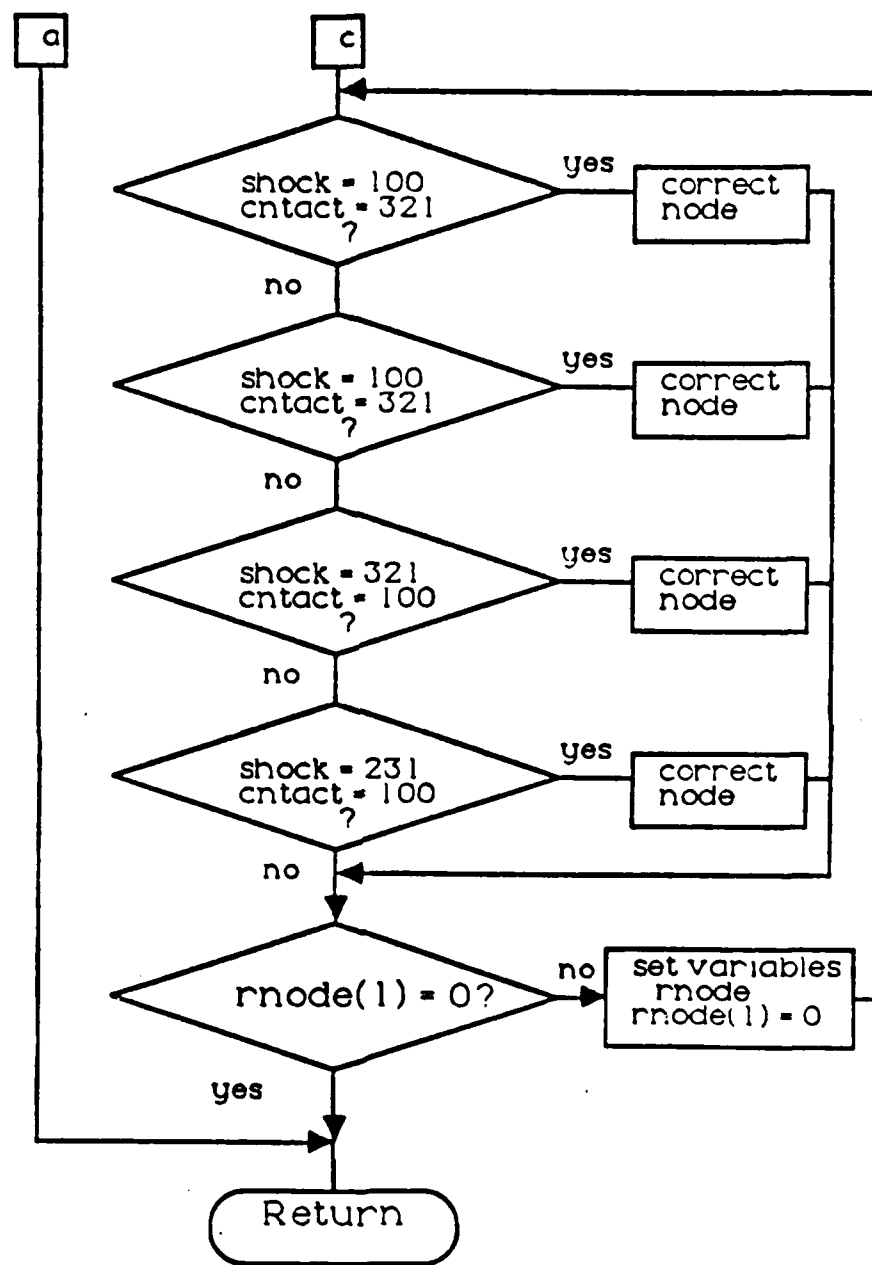


Figure C.7 (Continued)

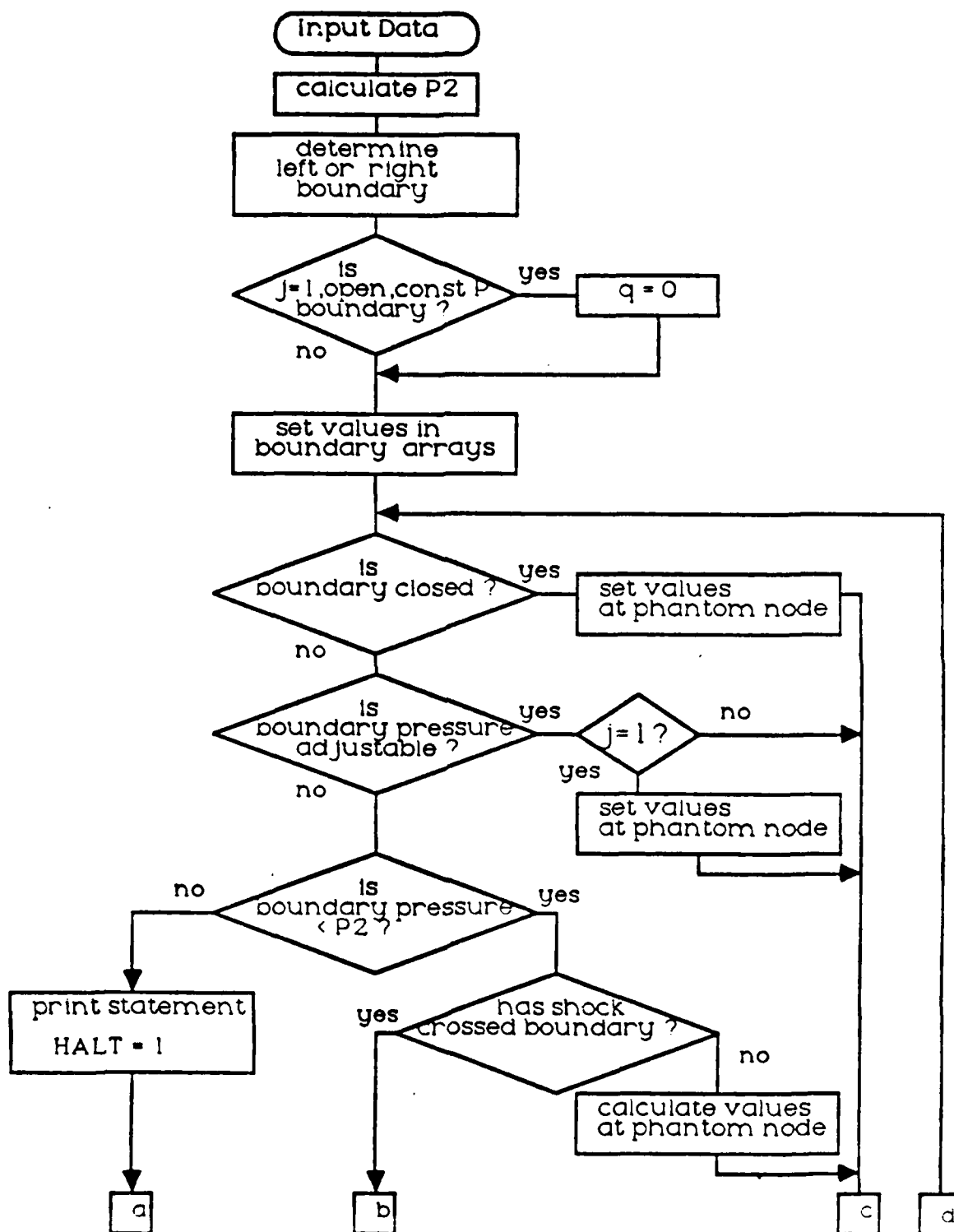


Figure C.8 "BONDY" Subroutine Flowchart

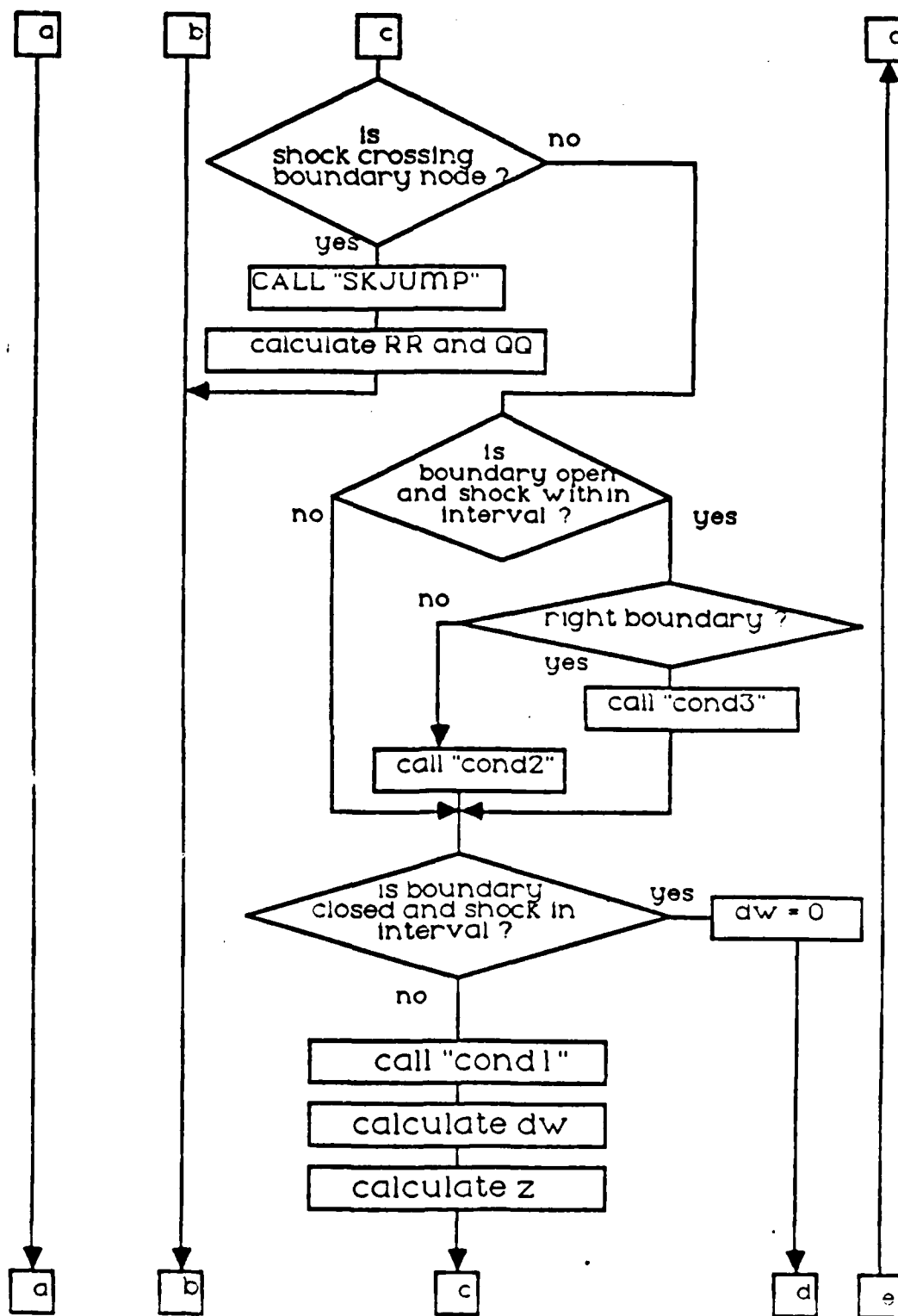


Figure C.8 (Continued)

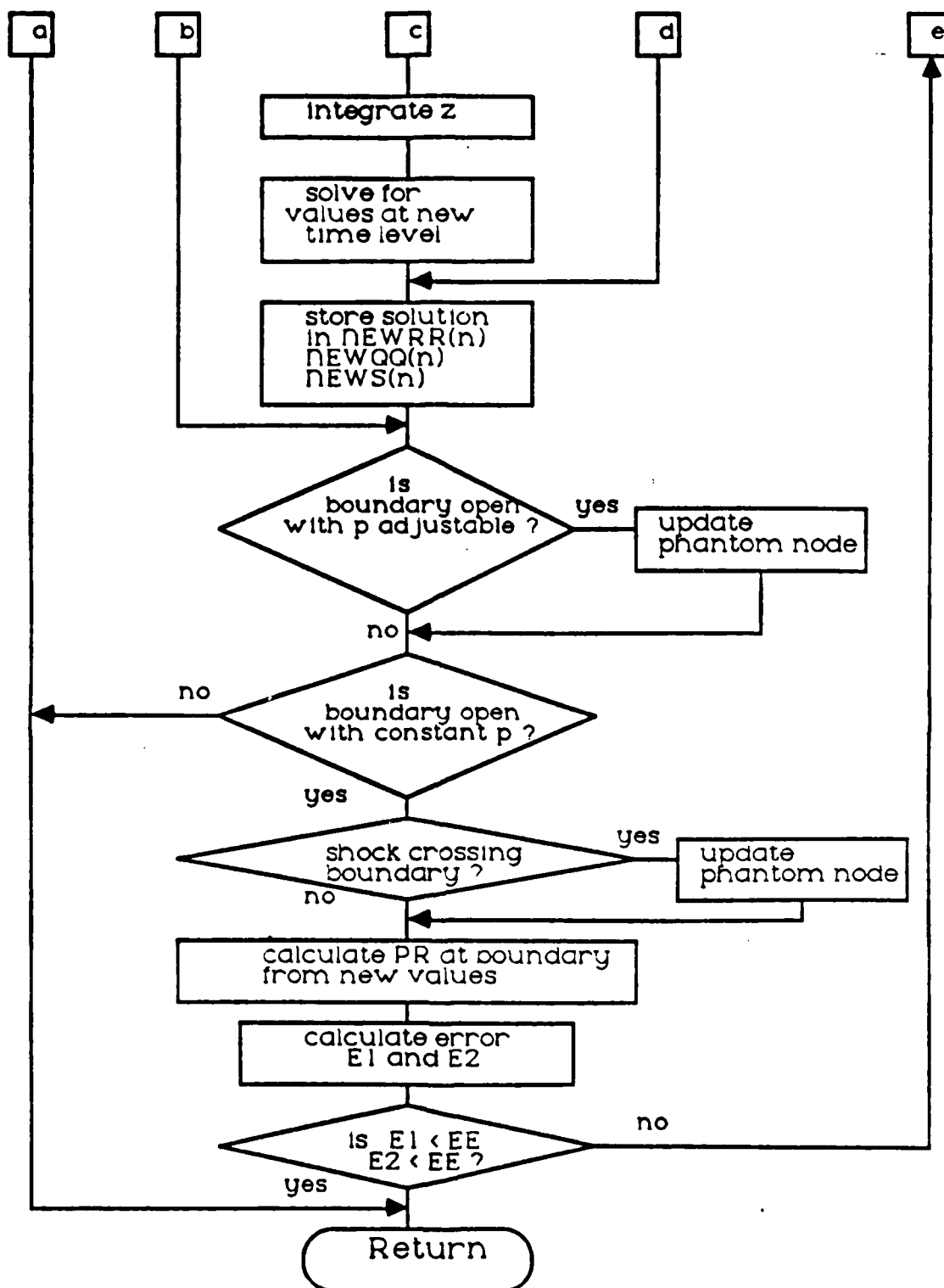


Figure C.8 (Continued)

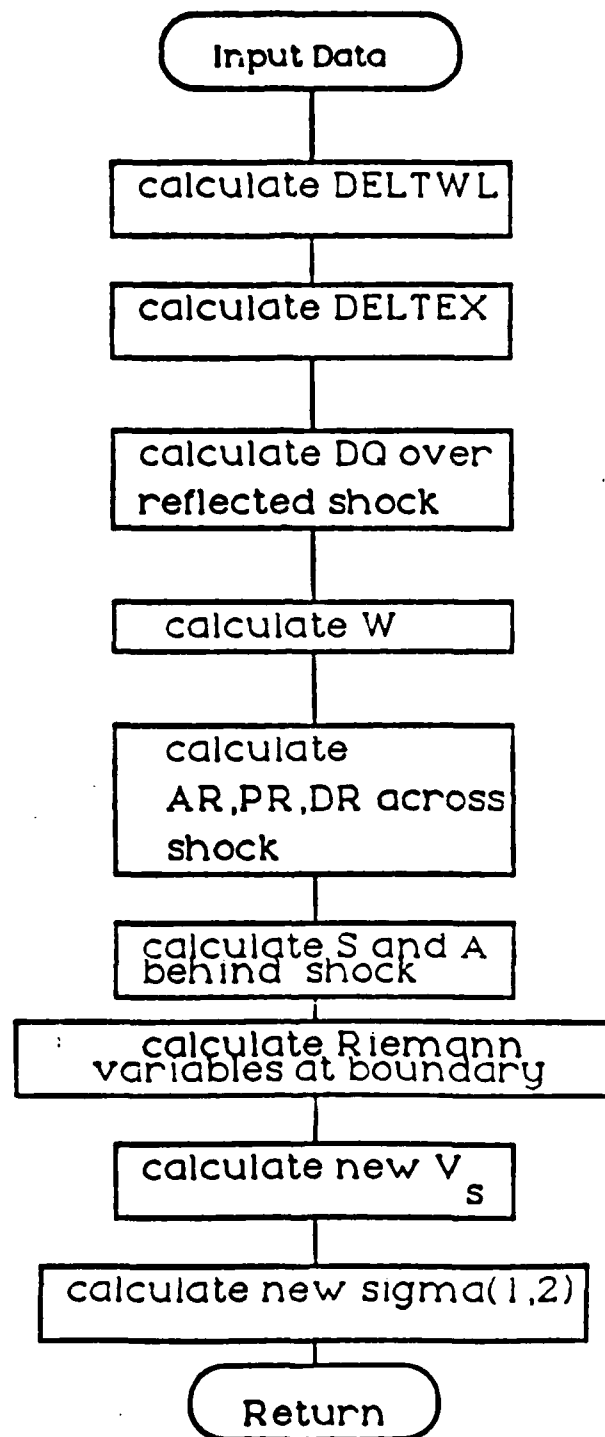


Figure C.9 "SRFLCT" Subroutine Flowchart

APPENDIX D

E1DV2 FORTRAN LISTING

```

C *****
C
C      *
C      *      E U L E R - 1 D
C      *      VERSION 2
C      *      (E1DV2)
C      *
C      *      THIS PROGRAM SOLVES THE EULER EQUATIONS
C      *      EXPRESSED IN A QUASI-ONE DIMENSIONAL
C      *      STREAMLINE COORDINATE SYSTEM.
C      *
C      *      AUTHOR - LT. D.T. JOHNSTON, FEB 1987
C      *
C      *      BASED ON THE EULER1 CODE BY
C      *      T.F. SALACKA, DEC 1985
C      *
C      *      FEATURES OF THIS VERSION (2)
C      *      + ORDER OF SPATIAL DERIVATIVES - FIRST
C      *      + NUMBER OF SPATIAL DIMENSIONS - ONE
C      *      + DISCONTINUITIES TREATED:
C      *          SHOCKS - YES
C      *          CONTACT DISCONTINUITIES - YES
C      *          EXPANSION WAVES - YES
C      *      + HIGH PRESSURE SIDE
C      *          LEFT - YES
C      *          RIGHT - YES
C      *
C      *
C      *****
C
C      ++++++
C      +
C      +      CONVENTIONS AND DEFINITIONS
C      +
C      +
C      ++++++
C
C      ----- NON-DIMENSIONING CONVENTION-----
C
C      ALL VELOCITIES NON-DIM. BY THE SOUND SPEED ON
C      THE LOW PRESSURE SIDE OF THE DIAPHRAGM.
C      ALL PRESSURES, DENSITIES, AND TEMPERATURES ARE
C      NON-DIM. BY THEIR INITIAL VALUES ON THE LOW
C      PRESSURE SIDE OF THE DIAPHRAGM.
C      SPACIAL DISTANCE IS NON-DIM. BY OVERALL LENGTH.
C      ENTROPY IS NON-DIM. BY THE GAS CONSTANT, R.
C      TIME IS NON-DIM. BY (LENGTH/SOUND SPEED).
C      VELOCITIES AND DISTANCES ARE DEFINED POSITIVE
C      TO THE RIGHT.
C
C      ----- SUBSCRIPT NOTATION-----
C
C      I - SPACIAL NODE (1 TO N FROM LEFT TO RIGHT)
C      J - TIME LEVEL (0 IS THE INITIAL CONDITION)
C      K - DENOTES WHICH CHARACTERISTIC WAVE IS BEING
C      DEALT WITH:
C          1 = Q+A    2 = Q    3 = Q-A
C      L - DENOTES WHICH TYPE OF DISCONTINUITY IS
C      BEING DEALT WITH:
C          1 = SHOCK    2 = CONTACT DISCONTINUITY
C          3 = HEAD OF EXPANSION WAVE
C          4 = TAIL OF EXPANSION WAVE
C
C

```

C
 C A - SPEED OF SOUND
 C *A - DENOTES THE VALUE OF A VARIABLE AT THE NODE
 C TO THE LEFT OF A DISCONTINUITY. * CAN
 C BE ANY VARIABLE NAME.
 C AR - THE RATIO OF SOUND SPEED ACROSS A
 C SHOCK, A/B (SHOCK MOVING RIGHT), B/A (SHOCK MOVING LEFT)
 C *B - DENOTES THE VALUE OF A VARIABLE AT THE NODE
 C TO THE RIGHT OF A DISCONTINUITY.
 C BDRY - 3 DENOTES LEFT BOUNDARY, 2 THE RIGHT BOUNDARY
 C COUNT - COUNTER FOR GRAPHICS ROUTINES
 C DARRAY - ARRAY OF DENSITY FOR PLOTTING
 C DELT - TIME STEP
 C DLCD - DENSITY BEHIND THE CONTACT
 C DISCONTINUITY IN THE EXACT SOLUTION.
 C DLSH - DENSITY BEHIND THE SHOCK IN THE
 C EXACT SOLUTION.
 C DQ - THE JUMP IN VELOCITY ACROSS THE SHOCK
 C DIVIDED BY THE SOUND SPEED AT B (RIGHT) OR A (LEFT)
 C DRI - INITIAL DENSITY RATIO ACROSS THE SHOCK
 C EE - DESIRED PRECISION FOR CHARACTERISTIC CALCULATIONS
 C G - GAMMA (RATIO OF SPECIFIC HEATS)
 C GRAPHS - FOR GRAPHICAL OUTPUT, 0=NONE (TABULAR)
 C 1=PLOTS ALL VARIABLES
 C 2=COMPARES DENSITY WITH EXACT SOLUTION
 C G1 - $1/(G-1)$
 C G2 - $2/(G-1)$
 C H - $1/(N-1)$
 C HALT - TERMINATES PROGRAM IF 1, SET BY CONDITIONS NOT CODED
 C I2 - NUMBER OF THE NODE TO THE RIGHT OF A
 C DISCONTINUITY.
 C JSTOP - NUMBER OF TIME LEVELS TO BE CALCULATED
 C LBDDR - LEFT BOUNDARY DENSITY RATIO
 C LBDDR1 - LEFT BOUNDARY DENSITY RATIO AT TIME ZERO
 C LBOPR - LEFT BOUNDARY PRESSURE RATIO
 C LBOPR1 - LEFT BOUNDARY PRESSURE RATIO AT TIME ZERO
 C LBOPRS - VALUE OF 0 DENOTES CONSTANT PRESSURE AT LEFT BOUNDARY
 C ,1 DENOTES ADJUSTABLE PRESSURE AT THE LEFT BOUNDARY
 C LBOTR - LEFT BOUNDARY TEMPERATURE RATIO
 C LBOTR1 - LEFT BOUNDARY TEMPERATURE RATIO AT TIME ZERO
 C LBNDRY - DENOTES LEFT BOUNDARY CONDITION, OPEN OR CLOSED
 C LNODE - ARRAY OF LEFT MOST NODE TO BE CORRECTED IN CORRCT
 C LWPRES - DENOTES WHICH SIDE OF DIAPHRAGM HAS LOW PRESSURE
 C N - NUMBER OF SPACIAL NODES (ODD NUMBER)
 C ND - DOUBLE PRECISION VALUE OF N
 C NEW**(I) - STORED VALUES OF ** FOR THE NEXT TIME LEVEL
 C PARRAY - ARRAY OF PRESSURES FOR PLOTTING
 C PRI - INITIAL PRESSURE RATIO ACROSS THE SHOCK
 C PLTCNT - COUNTER FOR GRAPHICS ROUTINES
 C Q - ABSOLUTE FLUID VELOCITY
 C QARRAY - ARRAY OF VELOCITIES FOR PLOTTING
 C QLBD - INITIAL VELOCITY AT LEFT BOUNDARY
 C QLI - INITIAL VELOCITY LEFT OF THE DIAPHRAGM
 C QRBD - INITIAL VELOCITY AT RIGHT BOUNDARY
 C QRI - INITIAL VELOCITY RIGHT OF THE DIAPHRAGM
 C QQ - $Q+A*S$ (EXTENDED RIEMANN VARIABLE)
 C RBDDR - RIGHT BOUNDARY DENSITY RATIO
 C RBDDR1 - INITIAL RIGHT BOUNDARY DENSITY RATIO
 C RBOPR - RIGHT BOUNDARY PRESSURE RATIO
 C RBOPR1 - INITIAL RIGHT BOUNDARY PRESSURE RATIO
 C RBOPRS - VALUE OF 0 DENOTES A CONSTANT PRESSURE AT RIGHT
 C BOUNDARY, WHILE 1 IS FOR ADJUSTABLE PRESSURE
 C RBOTR - RIGHT BOUNDARY TEMPERATURE RATIO
 C RBOTR1 - INITIAL RIGHT BOUNDARY TEMPERATURE RATIO
 C RBNDRY - DENOTES RIGHT BOUNDARY OPEN OR CLOSED
 C RNODE - ARRAY FOR RIGHT MOST NODE TO BE CORRECTED IN CORRCT
 C RR - $Q-A*S$ (EXTENDED RIEMANN VARIABLE)
 C S - ENTROPY
 C SARRAY - ARRAY OF ENTROPY FOR PLOTTING

```

C   SIGMA - SPATIAL LOCATION OF DISCONTINUITIES
C           SIGMA(L,J) WHERE L INDICATES THE TYPE OF
C           DISCONTINUITY AND J INDICATES THE
C           TIME LEVEL, 1 - CURRENT LEVEL
C           2 - LEVEL BEING CALCULATED
C   SK - INTEGER THAT DENOTES RELATIVE LOCATION OF SHOCK NEAR
C         BOUNDARIES
C   SKIP - VARIABLE WHICH INDICATES HOW MANY TIME STEPS BETWEEN
C           CALLS TO OUTPUT ROUTINES
C   T - TIME SINCE INITIAL CONDITIONS
C   TRI - INITIAL TEMP. RATIO ACROSS THE SHOCK
C   VHEAD - VELOCITY OF THE HEAD OF THE EXPANSION
C           WAVE FOR THE EXACT SOLUTION.
C   VTAIL - VELOCITY OF THE TAIL OF THE EXPANSION
C           WAVE FOR THE EXACT SOLUTION.
C   VCDE - VELOCITY OF THE CONTACT DISCONTINUITY
C           FOR THE EXACT SOLUTION.
C   VS - THE SHOCK SPEED(POSITIVE RIGHT, NEGATIVE LEFT)
C   VSE - VELOCITY OF THE SHOCK FOR THE EXACT
C           SOLUTION.
C   W - MACH NO. RELATIVE TO A STANDING SHOCK
C   XARRAY - ARRAY OF SPATIAL POSITIONS FOR PLOTTING
C   XEXACT - ARRAY OF SIX X VALUES FOR THE EXACT SOLUTION.
C   XINIT - INITIAL POSITION OF DISCONTINUITY FOR
C           EXACT SOLUTION PLOTTING.
C   X2 - LOCATION OF NODE TO RIGHT OF DISCONT.
C         ALONG THE SPACIAL AXIS.
C   Y - (N+1)/2
C   YEXACT - ARRAY OF SIX DENSITY VALUES FOR THE EXACT SOLUTION.
C
C   *** OTHER VARIABLES ARE DEFINED IN THE
C         SUBROUTINES WHERE THEY ARE USED ***
C
C   ++++++
C   +               PROBLEM SET - UP               +
C   ++++++
C
C   THE PARTICULAR PROBLEM FOR THIS VERSION IS:
C
C       SHOCK TUBE, SINGLE CENTERED DIAPHRAGM WITH
C       HIGH PRESSURE SIDE TO THE RIGHT.
C
C       BOUNDARY CONDITIONS - LEFT END CLOSED,RIGHT END OPEN
C
C   ----- VARIABLE DECLARATIONS -----
C
C       DIMENSION I2(4),X2(4),XEXACT(6),YEXACT(6),LNODE(4),RNODE(4),
C           SIGMA(4,2)
C
C   ++++++ USER INPUT REQUIRED HERE ++++++
C
C   ----- SET THE DIMENSIONS EQUAL TO N -----
C
C       DIMENSION A(101),Q(101),QQ(101),RR(101),S(101),
C           NEWRR(101),NEWS(101),NEWQQ(101),
C           PARRAY(101),DARRAY(101),SARRAY(101),
C           QARRAY(101),XARRAY(101)
C
C       INTEGER I,J,N,JSTOP,Y,GRAPHS,COUNT,PLTCNT,BDRY,SK,RBNDRY,
C           SKIP,I2,LWPRES,HALT,LNODE,RNODE,LBNDRY,LBOPRS,RBOPRS
C       DOUBLE PRECISION TRI,PRI,QLI,QRI,DRI,G,G1,G2,SIGMA,EE,NEWQQ,
C           DELT,H,ND,X2,AR,W,DQ,VS,T,A,Q,QQ,RR,S,NEWRR,NEWS,
C           LBOPRI,LBOTRI,LBODRI,LBOPR,LBODR,LBOTR,QLBD,
C           RRA,QQA,AA,SA,QA,RRB,QQB,AB,SB,QB,
C           RBOPRI,RBOTRI,RBODRI,RBOPR,RBODR,RBOTR,QRBD,
C           QCS,VTEM,VHEM
C       REAL VTAIL,VCDE,VSE,DLSH,DLCD,XINIT,VHEAD,

```

```

C      XEXACT,YEXACT,PARRAY,DARRAY,SARRAY,GARRAY,XARRAY
COMMON AR,DQ,VS,M
C
C      ----- ENTER THE APPROPRIATE VALUES BELOW -----
C
C      N= 101
C      GRAPHS=1
C
C      ----- FOR GRAPH = 1 OR 2 MUST ENTER CHANGES IN SUBROUTINE -----
C      ----- BORDER AND SUBROUTINE EXACT -----
C      ----- LINES "FIRST ORDER N = ???"
C      ----- "DENSITY RATIO = ??? TEMP RATIO = ???"
C      ----- "PRESSURE RATIO = ???"
C
C      SKIP=18
C      JSTOP=101
C      TRI=1.0000
C      PRI=5.0000
C      DRI=5.0000
C      QLI=0.0000
C      QRI=0.0000
C      LBDPRI = 1.000
C      LBDTRI = 1.000
C      LBDORI = 1.000
C      RBDORI = 1.000
C      RBDPRI = 4.000/5.000
C      RBDTRI = 1.000
C      G=1.4000
C      EE=0.1D-8
C
C      ----- DENOTE LOW PRESSURE SIDE BY SETTING -----
C      ----- LWPRES = 2 IF LOW PRESSURE ON RIGHT
C      ----- LWPRES = 3 IF LOW PRESSURE ON LEFT
C
C      LWPRES = 3
C
C      ----- SET BOUNDARY CONDITIONS BY SPECIFYING OPEN OR CLOSED -----
C      ----- LBNDRY: OPEN = 0, CLOSED = 1 (FOR LEFT BOUNDARY) -----
C      ----- IF OPEN SPECIFY IF PRESSURE IS TO BE MAINTAINED AT LBDPRI -----
C      ----- OR IF IT CAN ADJUST TO PREVENT ANY WAVES FORMING AT THE BOUNDARY -
C      ----- LBDPRS: CONSTANT = 0, ADJUSTABLE = 1 -----
C      ----- DO THE SAME FOR RIGHT BOUNDARY; RBNDRY,RBOPRS -----
C
C      LBNDRY = 1
C      LBDPRS = 1
C      RBNDRY = 0
C      RBDPRS = 0
C
C      ----- EXACT SOLUTION VALUES -----
C      XINIT=0.50
C      VHEAD= 1.0
C      VTAIL= 0.310557
C      VCDE=-.574487
C      VSE=-1.402346
C      DLCD=2.713115
C      DLSH=1.69344
C      SIGMA(1,2)=0.50000001000
C      SIGMA(2,2)=0.50000001000
C      SIGMA(3,2)=0.50000001000
C      SIGMA(4,2)=0.50000001000
C
C      ++++++ END OF USER INPUT AREA ++++++
C
C      SK = 0
C      T=0.0000
C      DO 10 I=1,4
C          I2(I)=0
C          LNODE(I) = 0
C          RNODE(I) = 0
C          X2(I)=0.0000

```



```

        SIGMA(I,1)=0.0000
10 CONTINUE
        ND=0BLE(N)
        H=1.000/(ND-1.000)
        DO 11 I=1,N
            A(I)=0.0000
            Q(I)=0.0000
            NEWQQ(I)=0.0000
            NEWRR(I)=0.0000
            NEKS(I)=0.0000
            QARRAY(I)=0.0
            PARRAY(I)=0.0
            DARRAY(I)=0.0
            SARRAY(I)=0.0
            XARRAY(I)=FLOAT(I-1)*SNGL(H)
11 CONTINUE
        DELT=2.0000
C
C -----LOAD INITIAL REIMAN VALUES INTO NODE LOCATIONS,FIRST FROM NODE---
C -----1 TO MIDPOINT (Y), AND THEN FROM Y TO N. NOTE IF SHOCK DOES NOT--
C -----START AT MIDPOINT THEN Y SHOULD BE SET TO NODE WHERE SHOCK -----
C -----INITIALLY IS-----
C
        AR=1.0000
        DQ=0.0000
        M=1.0000
        VS=0.0000
        QCS=0.000
        VHEW=0.000
        VTEW=0.000
        G1=1.000/(G-1.000)
        G2=2.000/(G-1.000)
C
C ----- FLOW RIGHT -----
C
        IF(LMPRES.EQ.2) THEN
            LBDDR = DRI * LBDDR
            LBOPR = PRI * LBOPRI
            LBOTR = TRI * LBOTRI
            QLBD = QLI
            RBDOR = RBDORI
            RBOPR = RBDPRI
            RBOTR = RBDTRI
            QRBO = QRI
            Y = (N+1)/2
C
C Y = 38
            DO 12 I=1,Y
                S(I)=G2-(G1/G)*DLOG(PRI/((DRI)**G))
                QQ(I)=QLI+OSQRT(TRI)*S(I)
                RR(I)=QLI-OSQRT(TRI)*S(I)
12 CONTINUE
                Y=(N+3)/2
C
C Y=39
            DO 13 I=Y,N
                S(I)=G2
                QQ(I)=QRI+S(I)
                RR(I)=QRI-S(I)
13 CONTINUE
            ELSE
C
C ----- FLOW LEFT -----
C
            LBDDR = LBDDR
            LBOPR = LBOPRI
            LBOTR = LBOTRI
            QLBD = QLI
            RBDOR = RBDORI * DRI
            RBOPR = RBDPRI * PRI
            RBOTR = RBDTRI * TRI

```

```

      QRBD = QRI
      Y = (N-1)/2
C     Y = 13
      DO 14 I=1,Y
          S(I)=G2
          QQ(I)=QLI+S(I)
          RR(I)=QLI-S(I)
14    CONTINUE
      Y = (N+1)/2
C     Y = 14
      DO 15 I=Y,N
          S(I)=G2-(G1/G)*DLOG(PRI/((DRI)**G))
          QQ(I)=QRI+DSQRT(TRI)*S(I)
          RR(I)=QRI-DSQRT(TRI)*S(I)
15    CONTINUE
      END IF
C
C ----- SET UP GRAPHICS PLOTS OF VARIABLES -----
C
      IF (GRAPHS.GT.0) THEN
          CALL COMPRS
C     CALL TEK618
          CALL HWROT('AUTO')
          CALL HMSCAL('SCREEN')
          IF (GRAPHS.EQ.2) THEN
              CALL PAGE(11.0,8.5)
          ELSE
              CALL PAGE(8.5,11.0)
          END IF
          IF (GRAPHS.EQ.1) THEN
              CALL BORDER(JSTOP)
          END IF
      END IF
      HALT = 0
      J=1
      COUNT=1
      IF (GRAPHS.EQ.1) THEN
          CALL PLOT(J,JSTOP,N,QQ,RR,S,H,XARRAY,PARRAY,
#DARRAY,QARRAY,SARRAY,G,G1,G2)
      END IF
C
C ----- BURST DIAPHRAGM -----
C
      CALL TIME(N,QQ,RR,S,DELT,H)
      CALL DBURST(N,H,QQ,RR,S,G,G1,G2,DELT,I2,X2,W,AR,DQ,VS,LWPRES,
C          SIGMA,A,Q)
C
      IF (GRAPHS.EQ.0) THEN
          CALL LIST(N,SIGMA,QQ,RR,S,G,G1,G2,J,T,DELT,VS,QCS,VTEW,VHEW,
C          XINIT,VHEAD,VTAIL,VCDE,VSE)
      END IF
C
C ----- BEGIN CALCULATION FOR JUMP TO NEXT TIME AND CONTINUE -----
C ----- UNTIL EITHER JSTOP REACHED OR SHOCK MEETS CONTACT SURFACE -----
C
16  IF (J.EQ.JSTOP) GOTO 18
      PLTCNT=J/SKIP
C
      CALL TIME(N,QQ,RR,S,DELT,H)
C
      CALL TRAK(N,SIGMA,H,QQ,RR,S,G,G1,G2,DELT,I2,X2,W,AR,DQ,VS,J,
C          LWPRES,QCS,VHEW,VTEW)
C
      CALL SWEET(N,H,SIGMA,QQ,RR,S,DELT,EE,Q,A,NEWQQ,NEWRR,NEWS,I2,G2,
C          J,LNODE,RNODE,HALT,LBNDRY,LBOPRS,LBDPR,LBDTR,LBDOR,
C          QLBD,G,G1,RBNDRY,RBOPRS,RBOPR,RBDTR,RBODR,QRBD,BDRY,
C          SK)
C
      IF (LNODE(4).LT.J) THEN

```

```

      LNODE(1) = 0
      RNODE(1) = 0
      GO TO 17
    END IF
    IF (RNODE(4).LT.LNODE(4)) THEN
      RNODE(1) = 0
    END IF
  C
    CALL CORRECT(LNODE,RNODE,N,SIGMA,H,QQ,RR,S,G,G1,G2,I2,X2,M,AR,DQ,
  C      VS,A,Q)
  C
17 IF (SIGMA(1,2).GE.1.D00) THEN
      IF (RBNDRY.EQ.0) THEN
        IF (SIGMA(1,2).NE.3.D00) THEN
          SK = 2
          CALL BONDY(Q(N),Q(N-1),QRBD,A(N),A(N-1),QQ(N),QQ(N-1),
  C      RR(N),RR(N-1),S(N),S(N-1),H,EE,DELT,
  C      RBNDY,RBOPRS,RBOPR,RBODR,RBOTR,J,NEWQQ(N),
  C      NEWRR(N),NEWS(N),G,G1,G2,HALT,BDRY,SK)
        END IF
      ELSE
        CALL EXTRAP(RR(N-1),RR(N-2),QQ(N-1),QQ(N-2),S(N-1),
  C      S(N-2),H,H,RRR,QQQ,SA,AA,QA)
        CALL SRFLCT(QQA,RRR,SA,SIGMA,VS,DELT,LWPRES,
  C      RR(N),QQ(N),S(N),Q(N),A(N),G,G1,G2)
      END IF
    END IF
  C
    IF (SIGMA(1,2).LE.0.D00) THEN
      IF (LBNDRY.EQ.0) THEN
        IF (SIGMA(1,2).NE.-2.D00) THEN
          BDRY = 3
          SK=2
          CALL BONDY(Q(1),Q(2),QLBD,A(1),A(2),QQ(1),QQ(2),
  C      RR(1),RR(2),S(1),S(2),H,EE,DELT,
  C      LBNDY,LBOPRS,LBOPR,LBODR,LBOTR,J,NEWQQ(1),
  C      NEWRR(1),NEWS(1),G,G1,G2,HALT,BDRY,SK)
        END IF
      ELSE
        CALL EXTRAP(RR(2),RR(3),QQ(2),QQ(3),S(2),
  C      S(3),H,H,RRR,QQB,SB,AB,QB)
        CALL SRFLCT(QQB,RRR,SB,SIGMA,VS,DELT,LWPRES,
  C      RR(1),QQ(1),S(1),Q(1),A(1),G,G1,G2)
      END IF
    END IF
    T=T+DELT
  C
  C ----- OUTPUT DATA -----
  C
    IF ((COUNT.EQ.PLCNT*SKIP).AND.(GRAPHS.EQ.1))
  C      THEN
  C    IF ((J.GT.55)) THEN
      CALL PLOT(J,JSTOP,N,QQ,RR,S,H,XARRAY,PARRAY,
  C      CDARRAY,QARRAY,SARRAY,G,G1,G2)
    END IF
    END IF
    IF ((COUNT.EQ.PLCNT*SKIP).AND.(GRAPHS.EQ.0))
  C      THEN
        CALL LIST(N,SIGMA,QQ,RR,S,G,G1,G2,J,T,DELT,VS,QCS,VTEM,VHEM,
  C      XINIT,VHEAD,VTAIL,VCDE,VSE)
    END IF
    IF ((COUNT.EQ.PLCNT*SKIP).AND.(GRAPHS.EQ.2))
  C      THEN
        IF (J.GT.50) THEN
          CALL EXACT(N,XINIT,T,VHEAD,VTAIL,VCDE,VSE,DLCD,DLSH,QQ,RR,S,H,
  C      CXARRAY,DARRAY,G,G1,G2,DRI)
        END IF
      END IF
    END IF
  C

```

```

      IF(HALT.EQ.1) GO TO 18
      J=J+1
      COUNT=COUNT+1
      GO TO 16
18  CALL DONEPL
      END
C *****
C *****
C *****
C
      SUBROUTINE LIST(N,SIGMA,QQ,RR,S,G,G1,G2,J,T,DELT,VS,QCS,VTEM,
C          VHEM,XINIT,VHEAD,VTAIL,VCDE,VSE)
C
C *****
C +
C +          TABULAR RESULTS SUBROUTINE          +
C +
C *****
C
C ----- VARIABLE DEFINITIONS -----
C
C DENS - DENSITY
C PRESS - PRESSURE
C TEMP - TEMPERATURE
C
      INTEGER I,J,N,L
      DIMENSION SIGMA(4,2),QQ(N),RR(N),S(N)
      DOUBLE PRECISION SIGMA,QQ,RR,S,PRESS,VTEM,VHEM,QCS,TEMPE,
C          TEMP,DENS,G,G1,G2,G,T,DELT,VS,HEMKE,
C          XINIT,VHEAD,VTAIL,VCDE,VSE,SKXE,CSXE
C
C      WRITE(9,*) 'TIME LEVEL',J,'          ELAPSED TIME IS',T
      WRITE(9,*) 'TIME STEP IS',DELT,'          SHOCK VELOCITY IS',VS
      WRITE(9,*) 'CONTACT SURFACE VELOCITY IS',QCS
      WRITE(9,*) 'HEAD EXPANSION WAVE VELOCITY IS',VHEM
      WRITE(9,*) 'TAIL EXPANSION WAVE VELOCITY IS',VTEM
      WRITE(9,*) ' '
      WRITE(9,*) '      NODE          VELOCITY          DENSITY
C PRESSURE'
      WRITE(9,*) ' '
      DO 61 I=1,N
      TEMP=(QQ(I)-RR(I))*(QQ(I)-RR(I))/(4.000*S(I)*S(I))
      DENS=((1.000/TEMP)*DEXP(G*(1.000-G)*(S(I)-G2)))*(-G1)
      PRESS=TEMP*DENS
      Q=(QQ(I)+RR(I))/2.0000
      WRITE (9,65) I,Q,DENS,PRESS
65  FORMAT (4X,I2,7X,F12.6,7X,F12.6,7X,F12.6)
61  CONTINUE
      WRITE(9,*) ' '
      WRITE(9,*) ' '
      WRITE(9,*) '      NODE          QQ          RR
C MODIFIED S'
      WRITE(9,*) ' '
      DO 62 I=1,N
      WRITE (9,66) I,QQ(I),RR(I),S(I)
66  FORMAT (4X,I2,7X,F12.6,7X,F12.6,7X,F12.6)
62  CONTINUE
      WRITE(9,*) ' '
      WRITE(9,*) ' '
      WRITE(9,*) '          DISCONTINUITY LOCATIONS AT TIME LEVEL',J
      WRITE(9,*) ' '
      WRITE(9,*) '          TYPE          LOCATION'
      DO 63 L=1,4
      WRITE(9,*) L,'          ',SIGMA(L,2)
63  CONTINUE
      WRITE(9,*) ' '
      WRITE(9,*) '-----'
C-----
      WRITE(9,*) ' '

```

```

      WRITE(9,*) ' '
      IF(J.EQ.1) THEN
        WRITE(10,*) '      TIME          SHOCK          CONTACT          HEAD
C   TAIL'
        WRITE(10,*) ' '
        END IF
        WRITE(10,67) T,SIGMA(1,1),SIGMA(2,1),SIGMA(3,1),SIGMA(4,1)
67  FORMAT (1X,F12.6,1X,F12.6,1X,F12.6,1X,F12.6,1X,F12.6)
      IF(J.EQ.1) THEN
        WRITE(8,*) '      EXACT VALUES'
        WRITE(8,*) '      TIME          SHOCK          CONTACT          HEAD
C   TAIL'
        WRITE(8,*) ' '
        END IF
        SKXE=XINIT+VSE*T
        CSXE=XINIT+VCOE*T
        HEMOE=XINIT+VHEAD*T
        TEMOE=XINIT+VTAIL*T
        WRITE(8,68) T,SKXE,CSXE,HEMOE,TEMOE
68  FORMAT (1X,F12.6,1X,F12.6,1X,F12.6,1X,F12.6,1X,F12.6)
      RETURN
      END

```

```

C
C   SUBROUTINE TIME(N,QQ,RR,S,DELT,H)
C
C *****
C *
C *   CALCULATE TIME STEP SUBROUTINE
C *
C *****
C
C -----NEW VARIABLE DEFINITIONS -----
C
C TMIN - RUNNING VALUE OF THE MINIMUM TIME STEP
C

```

```

      INTEGER N,I
      DIMENSION QQ(N),RR(N),S(N)
      DOUBLE PRECISION H,A,QQ,RR,S,DELT,TMIN,Q
      TMIN=2.0000
      DO 21 I=1,N
        A=(QQ(I)-RR(I))/(2.000*S(I))
        Q=(QQ(I)+RR(I))/2.0000
        DELT=H/(DABS(DABS(Q)+A))
        IF (DELT.LT.TMIN) THEN
          TMIN=DELT
        END IF
21  CONTINUE
      DELT=0.99000*TMIN
      RETURN
      END

```

```

C
C   SUBROUTINE TRAK(N,SIGMA,H,QQ,RR,S,G,G1,G2,DELT,I2,X2,N,AR,DQ,VS,
C   *J,LMPRES,QCS,VHEM,VTEM)
C
C *****
C *
C *   DISCONTINUITY TRACKING SUBROUTINE
C *
C *****
C
C ----- VARIABLE DEFINITIONS -----
C
C CSDIR - CONTACT SURFACE DIRECTION, 2 TO THE RIGHT,3 TO THE LEFT
C CSRPN - RIEMANN VARIABLE CHANGE ACROSS A CONTACT SURFACE
C DR - THE RATIO OF THE DENSITY ACROSS A
C      SHOCK, B/A(RIGHT),B/A(LEFT)
C DREPN - DUMMY VARIABLE
C PR - THE RATIO OF THE PRESSURE ACROSS A
C      SHOCK, A/B(RIGHT),B/A(LEFT)

```

```

C     MREIMN - THE MEASURED JUMP IN QQ ACROSS THE SHOCK,
C           FROM A TO B.
C     EREIMN - THE JUMP IN QQ ACROSS THE SHOCK CALCULATED ANALYTICALLY
C           AS A FUNCTION OF W.
C     SAP - ENTROPY TO THE LEFT OF THE SHOCK FOR FLOWS RIGHT
C           OR ENTROPY TO THE RIGHT OF THE C.S. FOR FLOWS LEFT
C     SBP - ENTROPY TO THE RIGHT OF THE C.S. FOR FLOWS RIGHT
C           OR ENTROPY TO THE LEFT OF THE SHOCK FOR FLOWS LEFT
C     SHKDIR - SHOCK DIRECTION OF TRAVEL,3 TO THE LEFT,2 TO THE RIGHT
C     TS - TIME FOR SHOCK TO TRAVEL ONE INTERVAL
C     X - DISTANCE FROM LEFT BOUNDARY TO NODE
C
C     INTEGER N,I,Y,I2,L,J,SHKDIR,CSDIR,LWPRES
C     DIMENSION SIGMA(4,2),X2(4),RR(N),QQ(N),S(N),I2(4)
C     DOUBLE PRECISION SIGMA,X2,X,H,AB,SA,SB,AA,QA,QB,QQA,QQB,RAA,RRB,
C           RR,QQ,S,TS,W,DQ,AR,PR,G,G1,G2,VS,DELT,CSRMN,
C           Q,MREIMN,DREMN,EREIMN,WB,SA1,SA2,SAP,SBP
C           AM,QW,VHEM,VTEM,TIME,QCS
C
C     ++++++ LOCATING THE UPSTREAM NODE ++++++
C
C     DO 10 L=1,4
C       SIGMA(L,1)=SIGMA(L,2)
C       Y=0
C       X=0.000
C       I=1
11    IF (.NOT.(Y.EQ.0)) GOTO 10
C       IF (SIGMA(L,1).LT.X) THEN
C         X2(L)=X
C         I2(L)=I
C         Y=1
C       END IF
C       X=X+H
C       I=I+1
C       GOTO 11
10   CONTINUE
C
C     ----- IF SHOCK HAS LEFT AN OPEN BOUNDARY OUT OF THE TUBE THEN SET -----
C     ----- SHOCK TO NEUTRAL -----
C
C     IF (I2(1).GT.N) THEN
C       SIGMA(1,1) = 2.000
C       SIGMA(1,2) = 3.000
C       W = 1.000
C       VS = 1.000
C       DQ = 0.000
C       AR = 0.000
C       PR = 1.000
C       DR = 1.000
C       GO TO 150
C     ELSE IF (I2(1).LT.2) THEN
C       SIGMA(1,1) =-1.000
C       SIGMA(1,2) =-2.000
C       W = 1.000
C       VS =-1.000
C       DQ = 0.000
C       AR = 0.000
C       PR = 1.000
C       DR = 1.000
C       GO TO 150
C     END IF
C
C     ++++++ DETERMINING SHOCK SPEED ++++++
C
C     IF ((J.EQ.1).OR.(I2(1).EQ.2).OR.(I2(1).EQ.N)) THEN
C
C     -----AT TIME ZERO OR BOUNDARYS DETERMINE CORRECT SHOCK DIRECTION-----
C     -----SHKDIR = 3 IS A SHOCK HEADED LEFT, AND SHKDIR = 2 IS SHOCK-----
C     -----TRAVELING RIGHT-----

```

```

C
      IF(LMPRES.EQ.3) THEN
        SHKDIR = 3
        IF (J.EQ.1) THEN
          X2(1) = X2(1) - H
          I2(1) = I2(1) - 1
          X2(2) = X2(2) - H
          I2(2) = I2(2) - 1
        END IF
        GO TO 20
      ELSE
        SHKDIR = 2
      END IF
      GO TO 20

C
C -----IF SHOCK AND CONTACT SURFACE ARE NOT WITHIN THE SAME-----
C -----INTERVAL THEN NO CORRECTIONS ARE NEEDED IN CALCULATING-----
C -----THE REIMAN VARIABLE JUMP ACROSS THE SHOCK-----
C
      ELSE IF (I2(1).NE.I2(2)) THEN
        IF((SHKDIR.EQ.3).AND.(SIGMA(1,1).EQ.(X2(1)-H))) THEN
          X2(1) = X2(1) - H
          I2(1) = I2(1) - 1
        END IF
        GO TO 20

C
C -----IF SHOCK AND CONTACT SURFACE ARE WITHIN THE SAME INTERVAL-----
C -----THEN CORRECTIONS ARE REQUIRED TO DETERMINE SHOCK STRENGTH-----
C
      ELSE IF(SHKDIR.EQ.3) THEN

C
C -----SHOCK LOCATION RELATIVE TO THE CONTACT SURFACE FOR A SHOCK-----
C -----HEADED TO THE LEFT DETERMINES THE CORRECT VALUES FOR W AND VS--
C
        IF(SIGMA(1,1).GT.SIGMA(2,1)) THEN
          GO TO 111
        ELSE IF(SIGMA(1,1).EQ.(X2(1)-H)) THEN
          X2(1) = X2(1) - H
          I2(1) = I2(1) - 1
          X2(2) = X2(2) - H
          I2(2) = I2(2) - 1
        END IF
15      RRA=RR(I2(1)-1)
          RRB=RR(I2(1))
          QQA=QQ(I2(1)-1)
          QQB=QQ(I2(1))
          SA=S(I2(1)-1)

C
          QA =(QQA+RRA)/2.000
          QB =(QQB+RRB)/2.000
          AA =(QQA-RRA)/(2.000*SA)
          DQ =(QB-QA)/AA
          W = DSQRT((DQ**2)*(0.36000)+1.000) - (DQ*0.6000)
          DQ =(-1.000)*DQ
          GO TO 110

C
C -----FOR SHOCK HEADED RIGHT THE SAME PROCEDURE FOR CORRECTIONS ARE---
C -----FOLLOWED-----
C
      ELSE IF(SIGMA(1,1).LT.SIGMA(2,1)) THEN
        GO TO 111
      ELSE
17      RRA=RR(I2(1)-1)
          RRB=RR(I2(1))
          QQA=QQ(I2(1)-1)
          QQB=QQ(I2(1))
          SB=S(I2(1))
          QA =(QQA+RRA)/2.000
          QB =(QQB+RRB)/2.000

```

```

      AB = (QGB-RRB)/(2.000*SB)
      DQ = (QA-QB)/AB
      M = DSQRT((DQ**2)*(0.36000)+1.000) + (DQ*0.6000)
      GO TO 110

    END IF

C
C -----WITH NO SHOCK/CONTACT SURFACE INTERACTION THE JUMP IN REIMAN-----
C -----VARIABLES ARE DETERMINED WITHOUT INTERPOLATION OVER THE -----
C -----INTERVAL. MREIMN IS THE MEASURED JUMP. EREIMN IS THE ANALYTICAL--
C -----VALUE-----
C
    20 RRA=RR(I2(1)-1)
       RRB=RR(I2(1))
       QQA=QQ(I2(1)-1)
       QGB=QQ(I2(1))
       SA=S(I2(1)-1)
       SB=S(I2(1))
    21 AB=(QGB-RRB)/(2.000*SB)
       AA=(QQA-RRA)/(2.000*SA)
       IF(SHKDIR.EQ.3) THEN
          MREIMN = (RRB-RRA)/AA
          DREIMN = DABS(MREIMN)
       ELSE
          MREIMN = (QQA-QGB)/AB
          DREIMN = MREIMN
       END IF

C
C -----ITERATE FOR PROPER VALUE OF M USING THE QUADRATIC FIT OF THE-----
C -----REIMAN VARIABLE CHANGE WITH M CURVE. NOTE LEFT MOVING SHOCKS-----
C -----ARE USED IN THESE EQUATIONS SINCE RRB-RRA/AA=-(QQA-QGB/AB)-----
C
    100 MM= (3.0396408001-((DREIMN+2.7574000)/0.286337000))
       W=5.513294000-DSQRT(MM)
       DQ=2.000*(MM-1.000)/(W*(G+1.000))
       AR=DSQRT(2.000*(G-1.000)*(1.000+((G-1.000)*MMW/2.000))*
          (G*G2*MM-1.000))/(G+1.000)*W
    C
       PR=(2.000*G/(G+1.000))*MM-((G-1.000)/(G+1.000))
       DR=((G-1.000)*MM+2.000)/((G+1.000)*MM)
       EREIMN=DQ+(AR-1.000)*G2-(AR*G1/G)*DLOG(PR*(DR**G))
       IF (DABS(EREIMN-DABS(MREIMN)).LT.0.1D-5) GO TO 110
       DREIMN = (DABS(MREIMN) - EREIMN) + DREIMN
       GOTO 100

C
C -----SHOCK VELOCITY DEPENDS ON DIRECTION SHOCK IS TRAVELING -----
C -----LEFT IS < 0, AND RIGHT IS > 0 -----
C
    110 IF (J.EQ.1) THEN
       TIME = 0.000
       SA1 = (G1/G)*DLOG((2.000*G*(MM**2)-G+1.000)/(G+1.000))
       SA2 = G1*DLOG(((G-1.000)*(MM**2)+2)/((G+1.000)*(MM**2)))
       IF(SHKDIR.EQ.2) THEN
          SAP = SB - SA1 - SA2
          SBP = SAP
       ELSE
          SBP = SA - SA1 - SA2
          SAP = SBP
       END IF
    103 IF(SHKDIR.EQ.2) THEN
       CSRMN = ((DEXP((SBP-SA)/G2))*(SA)-(SBP))*AR
       ELSE
       CSRMN = ((DEXP((SAP-SB)/G2))*(SB)-(SAP))*AR
       END IF
       IF(SHKDIR.EQ.3) THEN
          MREIMN = ((RRB-RRA)/AA)+CSRMN
          DREIMN = DABS(MREIMN)
       ELSE
          MREIMN = ((QQA-QGB)/AB)-CSRMN
          DREIMN = MREIMN
       END IF

```



```

101      MM=(3.0396408001-((DREMN+2.7574000)/0.286337000))
          M=5.513294000-DSQRT(MM)
          DQ=2.000*(MM-1.000)/(M*(G+1.000))
          AR=DSQRT(2.000*(G-1.000)*(1.000+((G-1.000)*MM/2.000))*
C          (G+G2*MM-1.000))/((G+1.000)*M)
          PR=(2.000*G/(G+1.000))*MM-((G-1.000)/(G+1.000))
          DR=((G-1.000)*MM+2.000)/((G+1.000)*MM)
          EREIMN=DQ+(AR-1.000)*G2-(AR*G1/G)*DLOG(PR*(DR*G))
          IF (DABS(EREIMN-DABS(MREIMN)).LT.0.10-5) GO TO 102
          DREMN = (DABS(MREIMN) - EREIMN) + DREMN
          GOTO 101
102      SA1 = (G1/G)*DLOG(2.000*G*(MM*2)-G+1.000)/(G+1.000)
          SA2 = G1*DLOG(((G-1.000)*(MM*2)+2)/((G+1.000)*(MM*2)))
          IF(SHKDIR.EQ.2) THEN
              SAP = SB - SA1 - SA2
          ELSE
              SBP = SA - SA1 - SA2
          END IF
          IF (DABS(SAP-SBP).LT.0.10-5) GO TO 105
          IF(SHKDIR.EQ.2) THEN
              SBP = (SAP-SBP) + SBP
          ELSE
              SAP = (SBP-SAP) + SAP
          END IF
          GO TO 103
      END IF
105 IF(SHKDIR.EQ.3) THEN
          DQ = (-1.000)*DQ
          VS = ((RRA+QQA)*0.5000) - (M*AA)
      ELSE
          VS=(QQB+RRB)*0.5000+M*AB
      END IF
          TS=H/DABS(VS)
111 IF (TS.LT.DELT) THEN
          DELT=0.99000*TS
      END IF
          SIGMA(1,2)=VS*DELT+SIGMA(1,1)
C
C *****DETERMINE CONTACT SURFACE SPEED*****
C
      IF(J.EQ.1) THEN
          CSDIR = SHKDIR
      END IF
C
C ----- CONTACT SURFACE TRAVELING RIGHT -----
C
      150 IF(CSDIR.EQ.2) THEN
C
C -----CONTACT SURFACE MOVING RIGHT, CHECK FOR SHOCK IN INTERVAL-----
C -----AND CALCULATE SPEED OF CONTACT SURFACE AS APPROPRIATE-----
C
          IF(J.EQ.1) THEN
              QB =(QQB+RRB)/2.000
              QA = DQ*AB + QB
              CALL SKJUMP(AB,QB,SB,AR,DQ,VS,G,G1,M,AA,QA,SA)
              Q = QA
              GO TO 200
          END IF
          IF(X2(2).EQ.X2(1)) THEN
              IF(SIGMA(2,1).LE.SIGMA(1,1)) THEN
                  Q = (QQ(I2(2)-1) + RR(I2(2)-1)) / 2.000
              ELSE
                  Q = (QQ(I2(2)) + RR(I2(2))) / 2.000
              END IF
          ELSE
              QA = (QQ(I2(2)-1) + RR(I2(2)-1)) / 2.000
              QB = (QQ(I2(2)) + RR(I2(2))) / 2.000
              Q = (QA + QB)/2.000
          END IF

```

```

ELSE IF(CSDIR.EQ.3) THEN
C
C ----- CONTACT SURFACE TRAVELING LEFT -----
C
      IF(J.EQ.1) THEN
        QA =(QQA+RRA)/2.000
        QB = DQ*AA + QA
        CALL SKJUMP(AA,QA,SA,AR,DQ,VS,G,G1,H,AB,QB,SB)
        Q = QB
        GO TO 200
      END IF
      IF(X2(2).EQ.X2(1)) THEN
        IF(SIGMA(2,1).GE.SIGMA(1,1)) THEN
          Q = (QQ(I2(2)) + RR(I2(2))) / 2.000
        ELSE
          Q = (QQ(I2(2)-1) + RR(I2(2)-1)) / 2.000
        END IF
      ELSE IF(SIGMA(2,1).EQ.(X2(2)-H)) THEN
        X2(2) = X2(2) - H
        I2(2) = I2(2) - 1
        Q = (QQ(I2(2))+RR(I2(2))) / 2.000
      ELSE
        QA = (QQ(I2(2)-1) + RR(I2(2)-1)) / 2.000
        QB = (QQ(I2(2)) + RR(I2(2))) / 2.000
        Q = (QA + QB)/2.000
      END IF
      END IF
      200 SIGMA(2,2) = DELT * Q + SIGMA(2,1)
      QCS=Q
C
C ----- CALCULATE EXPANSION WAVE SPEED -----
C
      TIME = TIME+DELT
      IF(J.EQ.3) THEN
        AM=(QQ((N+1)/2)-RR((N+1)/2))/(2.000*S((N+1)/2))
        QM=Q
        IF((CSDIR).EQ.2) THEN
          VHEW=-(AM)
          VTEW= QM-AM
        ELSE
          VHEW= AM
          VTEW= QM+AM
        END IF
      END IF
      IF(J.EQ.3) THEN
        SIGMA(3,2) = SIGMA(3,1) + VHEW*TIME
        SIGMA(4,2) = SIGMA(4,1) + VTEW*TIME
      ELSE IF((CSDIR.EQ.2).AND.(SIGMA(3,1).GT.0.000)) THEN
        SIGMA(3,2) = SIGMA(3,1) + VHEW*DELT
        SIGMA(4,2) = SIGMA(4,1) + VTEW*DELT
      ELSE IF((CSDIR.EQ.3).AND.(SIGMA(3,1).GT.1.000)) THEN
        SIGMA(3,2) = SIGMA(3,1) + VHEW*DELT
        SIGMA(4,2) = SIGMA(4,1) + VTEW*DELT
      END IF
      RETURN
      END
C
      SUBROUTINE SWEEP(N,H,SIGMA,QQ,RR,S,DELT,EE,Q,A,NEWQQ,NEWRR,NEWS,
      CI2,G2,J,LNODE,RNODE,HALT,LBNDRY,LBOPRS,LBDPR,LBDTR,LBDDR,QLBD,G,
      CG1,RBNDRY,RBOPRS,RBOPR,RBDR,RBDDR,QRBD,BDRY,SK)
C
C *****
C *
C *      SPACE SWEEPING SUBROUTINE
C *
C *****
C
C ----- VARIABLE DEFINITIONS -----

```

```

C      AAVG - AVERAGE SPEED OF SOUND
C      CNTACT - 3 DIGIT VARIABLE DENOTING CONTACT SURFACE
C              LOCATION,DIRECTION OF TRAVEL, AND IF IT CROSSES A NODE
C      DELQQH - CHANGE IN QQ FROM I TO I+1
C      DELQQL - CHANGE IN QQ FROM I-1 TO I
C      DELRRH - CHANGE IN RR FROM I TO I+1
C      DELRRL - CHANGE IN RR FROM I-1 TO I
C      DELSH - CHANGE IN S FROM I TO I+1
C      DELSL - CHANGE IN S FROM I-1 TO I
C      DELAH - CHANGE IN A FROM I TO I+1
C      DELAL - CHANGE IN A FROM I-1 TO I
C      DELQH - CHANGE IN Q FROM I TO I+1
C      DELQL - CHANGE IN Q FROM I-1 TO I
C      DELX - INTERPOLATION DISTANCE (LMD*DELT)
C      DLTA** - PREFIX WHICH INDICATES THE SPATIAL
C              CHANGE IN ** FOR ONE TIME STEP.
C      INTEG(K)- RESULT OF INTEGRATING Z(K)
C      **INT - VALUE OF ** INTERPOLATED BETWEEN NODES
C              ON THE CURRENT TIME LEVEL.
C      LXX - NODE DEFINING THE LEFT INTERVAL
C      *PRIM(K)- SUFFIX WHICH INDICATES THE SPATIAL
C              DERIVATIVE OF * AT THE CURRENT TIME LEVEL.
C      RXX - NODE DEFINING THE RIGHT INTERVAL
C      SAVG - AVERAGE ENTROPY
C      SHOCK - 3 DIGIT VARIABLE DENOTING SHOCK
C              LOCATION,DIRECTION OF TRAVEL, AND IF IT CROSSES A NODE
C      **STEP - THE CHANGE IN TIME OF ** AT A NODE
C              USED TO STEP UP TO THE NEXT TIME LEVEL
C      X - LOCATION IN SPACIAL PLANE (I-1)*H
C      Z(K) - RIGHT SIDE OF THE K'TH EQUATION.
C
C      INTEGER I,RXX,LXX,SHOCK,CNTACT,I2,J,LNODE,RNODE,HALT,
C      N,LBNDY,LBOPRS,RBNDY,RBOPRS,SK,BORY
C      DIMENSION SIGMA(4,2),S(1),Q(1),A(1),I2(4),QINT(3),AINT(3),Z(3),
C      NEWQQ(1),NEWRR(1),NEWS(1),INTEG(3),APRIM(3),QPRIM(3),
C      LNODE(4),RNODE(4),AAVG(3),RR(1),QQ(1)
C      DOUBLE PRECISION AAVG,SAVG,G2,X,H,SIGMA,QQ,RR,S,Q,A,G,G1,
C      DELQQH,DELQQL,DELRRH,DELRRL,DELSH,DELSL,
C      DELAH,DELAL,DELQH,DELQL,DELT,
C      QINT,AINT,QQINT,RRINT,SINT,EE,
C      NEWQQ,NEWRR,NEWS,LBOPR,LBOTR,LBODR,QLBD,
C      RRSTEP,SSTEP,INTEG,Z,DLTAQQ,QQSTEP,
C      DLTARR,DLTAS,APRIM,QPRIM,
C      RBOPR,RBOTR,RBODR,QRBD
C      COMMON AR,DQ,VS,H
C
C      ----- COMPUTE VELOCITY AND SPEED OF SOUND AT EACH NODE -----
C
C      DO 10 I= 1,N
C          Q(I) = (QQ(I) + RR(I)) / 2.0000
C          A(I) = ((QQ(I) - RR(I)) / (2.0000 * S(I)))
C      10 CONTINUE
C
C      ----- ADVANCE LEFT BOUNDARY TO NEW TIME STEP -----
C
C      BORY = 3
C      IF(I2(1).EQ.2) THEN
C          SK = 1
C      ELSE IF(SIGMA(1,2).EQ.-2.000) THEN
C          SK = 3
C      ELSE
C          SK = 0
C      END IF
C      CALL BONDY(Q(1),Q(2),QLBD,A(1),A(2),QQ(1),QQ(2),RR(1),RR(2),
C      S(1),S(2),H,EE,DELT,LBNDY,LBOPRS,LBOPR,LBODR,LBOTR,
C      J,NEWQQ(1),NEWRR(1),NEWS(1),G,G1,G2,HALT,BORY,SK)
C
C      ----- AT EACH NODE FROM 2 TO N-1 DETERMINE THE BEST ALGORITHM TO USE--
C      ----- TO ADVANCE THAT NODE TO THE NEXT TIME STEP -----

```

```

C
  I=2
11 IF(I.EQ.N) GO TO 1200
    X=FLOAT(I-1)*H
    DELQQH = QQ(I+1) - QQ(I)
    DELQQL = QQ(I) - QQ(I-1)
    DELRRH = RR(I+1) - RR(I)
    DELRRL = RR(I) - RR(I-1)
    DELSH = S(I+1) - S(I)
    DELSL = S(I) - S(I-1)
    DELAH = A(I+1) - A(I)
    DELAL = A(I) - A(I-1)
    DELQH = Q(I+1) - Q(I)
    DELQL = Q(I) - Q(I-1)
C
C ----- DEFINE LEFT SECTOR AND RIGHT SECTOR WRT NODE EXAMINED-----
C
    RXX = I + 1
    LXX = I
C
C ----- TEST FOR SHOCK -----
C
    IF (I2(1).EQ.RXX) THEN
      SHOCK = 200
      GO TO 20
    ELSE IF (I2(1).EQ.LXX) THEN
      SHOCK = 300
      GO TO 20
    ELSE
      SHOCK = 100
    END IF
    GO TO 30
C
C ----- DETERMINE DIRECTION SHOCK IS TRAVELING -----
C
    20 IF (SIGMA(1,1).LT.SIGMA(1,2)) THEN
      SHOCK = SHOCK + 20
    ELSE
      SHOCK = SHOCK + 30
    END IF
C
C ----- DETERMINE IF SHOCK CROSSES A NODE IN THIS TIME INTERVAL -----
C
    IF (SHOCK.EQ.220) THEN
      IF (SIGMA(1,2).GE.(X+H)) THEN
        SHOCK = SHOCK + 1
      ELSE
        SHOCK = SHOCK + 2
      END IF
    ELSE IF (SHOCK.EQ.230) THEN
      IF (SIGMA(1,2).LE.X) THEN
        SHOCK = SHOCK + 1
      ELSE
        SHOCK = SHOCK + 2
      END IF
    ELSE IF (SHOCK.EQ.320) THEN
      IF (SIGMA(1,2).GE.X) THEN
        SHOCK = SHOCK + 1
      ELSE
        SHOCK = SHOCK + 2
      END IF
    ELSE IF (SHOCK.EQ.330) THEN
      IF (SIGMA(1,2).LE.(X-H)) THEN
        SHOCK = SHOCK + 1
      ELSE
        SHOCK = SHOCK + 2
      END IF
    END IF
C

```

```

C ----- TEST FOR CONTACT SURFACE -----
C
  30 IF (I2(2).EQ.RXX) THEN
      CNTACT = 200
      GO TO 40
  ELSE IF (I2(2).EQ.LXX) THEN
      CNTACT = 300
      GO TO 40
  ELSE
      CNTACT = 100
  END IF
  GO TO 50

C
C -----DETERMINE DIRECTION CONTACT SURFACE IS TRAVELING -----
C
  40 IF (SIGMA(2,1).LT.SIGMA(2,2)) THEN
      CNTACT = CNTACT + 20
  ELSE
      CNTACT = CNTACT + 30
  END IF

C
C ----- DETERMINE IF CONTACT SURFACE CROSSES A NODE DURING THIS TIME ---
C ----- INTERVAL -----
C
  IF (CNTACT.EQ.220) THEN
      IF (SIGMA(2,2).GE.(X+H)) THEN
          CNTACT = CNTACT + 1
      ELSE
          CNTACT = CNTACT + 2
      END IF
  ELSE IF (CNTACT.EQ.230) THEN
      IF (SIGMA(2,2).LE.X) THEN
          CNTACT = CNTACT + 1
      ELSE
          CNTACT = CNTACT + 2
      END IF
  ELSE IF (CNTACT.EQ.320) THEN
      IF (SIGMA(2,2).GE.X) THEN
          CNTACT = CNTACT + 1
      ELSE
          CNTACT = CNTACT + 2
      END IF
  ELSE IF (CNTACT.EQ.330) THEN
      IF (SIGMA(2,2).LE.(X-H)) THEN
          CNTACT = CNTACT + 1
      ELSE
          CNTACT = CNTACT + 2
      END IF
  END IF

C
C ---CHECK IF EITHER A SHOCK OR CONTACT SURFACE WITHIN H OF THIS NODE---
C ---DETERMINE PROPER ALGORITHM TO USE FOR CALCULATING EXTENDED REIMAN--
C ---VARIABLE CHANGE ALONG CHARACTERISTICS AT THIS NODE-----
C
  50 IF (SHOCK.EQ.100.OR.CNTACT.EQ.100) THEN
      C
      C ---NEITHER A SHOCK NOR A CONTACT SURFACE EXIST NEAR NODE---
      C
      IF (SHOCK.EQ.100.AND.CNTACT.EQ.100) THEN
          C
          C ---TEST FOR SUBSONIC OR SUPERSONIC FLOW---
          C
          IF (DABS(Q(I)).LT.A(I)) THEN
              IF (I.EQ.(I2(1)-2)) THEN
                  IF (J.EQ.1) THEN
                      IF (SIGMA(1,1).LE.SIGMA(1,2)) THEN
                          GO TO 200
                      END IF
                  END IF
              END IF
          END IF
      END IF
  END IF

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```

        END IF
        IF(I.EQ.(I2(1)+1)) THEN
            IF(J.EQ.1) THEN
                IF(SIGMA(1,1).GE.SIGMA(1,2)) THEN
                    GO TO 300
                END IF
            END IF
        END IF
        GO TO 100
    ELSE
        IF (SIGMA(2,1).LE.SIGMA(2,2)) THEN
            GO TO 200
        ELSE
            GO TO 300
        END IF
    END IF
END IF
END IF

C
C ---SHOCK OR CONTACT SURFACE ON LEFT, HEADED RIGHT, NO NODES CROSSED---
C
    IF (SHOCK.EQ.322.OR.CNTACT.EQ.322) THEN
        IF (DABS(Q(I)).LT.A(I)) THEN
            GO TO 300
        ELSE
            GO TO 500
        END IF
    END IF

C
C ---SHOCK OR CONTACT SURFACE ON LEFT, HEADED LEFT, NO NODES CROSSED---
C
    IF (SHOCK.EQ.332.OR.CNTACT.EQ.332) THEN
        GO TO 300
    END IF

C
C ---SHOCK OR CONTACT SURFACE ON LEFT, HEADED RIGHT, NODE IS CROSSED---
C
    IF (SHOCK.EQ.321.OR.CNTACT.EQ.321) THEN
        GO TO 400
    END IF

C
C ---SHOCK OR CONTACT SURFACE ON LEFT, HEADED LEFT, NODE IS CROSSED---
C
    IF (SHOCK.EQ.331.OR.CNTACT.EQ.331) THEN
        GO TO 300
    END IF

C
C ---SHOCK OR CONTACT SURFACE ON RIGHT, HEADED RIGHT, NO NODES CROSSED---
C
    IF (SHOCK.EQ.222.OR.CNTACT.EQ.222) THEN
        GO TO 200
    END IF

C
C ---SHOCK OR CONTACT SURFACE ON RIGHT, HEADED LEFT, NO NODES CROSSED---
C
    IF (SHOCK.EQ.232.OR.CNTACT.EQ.232) THEN
        IF (DABS(Q(I)).LT.A(I)) THEN
            GO TO 200
        ELSE
            GO TO 500
        END IF
    END IF

C
C ---SHOCK OR CONTACT SURFACE ON RIGHT, HEADED RIGHT, NODE CROSSED---
C
    IF (SHOCK.EQ.221.OR.CNTACT.EQ.221) THEN
        GO TO 200
    END IF

C
C ---SHOCK OR CONTACT SURFACE ON RIGHT, HEADED LEFT, JUMPS NODE---

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C      GO TO 400
      END IF
C
C ---BRANCH HERE IF SHOCK TO RIGHT OF CONTACT SURFACE-----
C ---DETERMINE PROPER ALGORITHM TO USE FOR CALCULATING EXTENDED REIMAN--
C ---VARIABLE CHANGE ALONG CHARACTERISTICS AT THIS NODE-----
C
      IF (SIGMA(1,1).GT.SIGMA(2,1)) THEN
C
C ---SHOCK ON LEFT, HEADED RIGHT, NO NODE JUMPED---
C
      IF (SHOCK.EQ.322) THEN
        IF (CNTACT.EQ.322) THEN
          IF (DABS(Q(I)).LT.A(I)) THEN
            GO TO 300
          ELSE
            GO TO 500
          END IF
        ELSE IF (CNTACT.EQ.332) THEN
          GO TO 800
        ELSE IF (CNTACT.EQ.331) THEN
          GO TO 800
        ELSE
          GO TO 900
        END IF
      END IF
C
C ---SHOCK ON LEFT, HEADED LEFT, NO NODE JUMPED---
C
      IF (SHOCK.EQ.332) THEN
        IF (CNTACT.EQ.322) THEN
          GO TO 300
        ELSE IF (CNTACT.EQ.332) THEN
          GO TO 600
        ELSE IF (CNTACT.EQ.331) THEN
          GO TO 600
        ELSE IF (CNTACT.EQ.321) THEN
          GO TO 700
        ELSE
          GO TO 900
        END IF
      END IF
C
C ---SHOCK ON LEFT, HEADED RIGHT, JUMPS NODE---
C
      IF (SHOCK.EQ.321) THEN
        IF (CNTACT.EQ.322.OR.CNTACT.EQ.321) THEN
          GO TO 400
        ELSE IF (CNTACT.EQ.332.OR.CNTACT.EQ.331) THEN
          GO TO 800
        ELSE
          GO TO 900
        END IF
      END IF
C
C ---SHOCK ON LEFT, HEADED LEFT, JUMPS NODE---
C
      IF (SHOCK.EQ.331) THEN
        IF (CNTACT.EQ.322.OR.CNTACT.EQ.321) THEN
          GO TO 700
        ELSE IF (CNTACT.EQ.332.OR.CNTACT.EQ.331) THEN
          GO TO 600
        ELSE
          GO TO 900
        END IF
      END IF
C
C ---SHOCK ON RIGHT, HEADED RIGHT, NO NODE JUMPED---

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```

C
  IF (SHOCK.EQ.222) THEN
    IF (CNTACT.EQ.222) THEN
      GO TO 200
    ELSE IF (CNTACT.EQ.232.OR.CNTACT.EQ.231) THEN
      GO TO 800
    ELSE IF (CNTACT.EQ.321.OR.CNTACT.EQ.322) THEN
      GO TO 400
    ELSE IF (CNTACT.EQ.332.OR.CNTACT.EQ.331) THEN
      GO TO 800
    ELSE
      GO TO 900
    END IF
  END IF
C
C ---SHOCK ON RIGHT, HEADED LEFT, NO NODE JUMPED---
C
  IF (SHOCK.EQ.232) THEN
    IF (CNTACT.EQ.222) THEN
      IF (SIGMA(1,2).LT.SIGMA(2,2)) THEN
        GO TO 700
      ELSE
        GO TO 200
      END IF
    ELSE IF (CNTACT.EQ.231.OR.CNTACT.EQ.232) THEN
      GO TO 600
    ELSE IF (CNTACT.EQ.221) THEN
      GO TO 700
    ELSE IF (CNTACT.EQ.322) THEN
      GO TO 400
    ELSE IF (CNTACT.EQ.332.OR.CNTACT.EQ.331) THEN
      GO TO 600
    ELSE IF (SIGMA(1,2).LT.SIGMA(2,2)) THEN
      GO TO 710
    ELSE
      GO TO 400
    END IF
  END IF
C
C ---SHOCK ON RIGHT, HEADED RIGHT, JUMPS NODE---
C
  IF (SHOCK.EQ.221) THEN
    IF (CNTACT.EQ.221.OR.CNTACT.EQ.222) THEN
      GO TO 200
    ELSE IF (CNTACT.EQ.231.OR.CNTACT.EQ.232) THEN
      GO TO 900
    ELSE IF (CNTACT.EQ.321.OR.CNTACT.EQ.322) THEN
      GO TO 400
    ELSE
      GO TO 800
    END IF
  END IF
C
C ---SHOCK ON RIGHT, HEADED LEFT, JUMPS NODE---
C
  IF (CNTACT.EQ.221.OR.CNTACT.EQ.222) THEN
    GO TO 700
  ELSE IF (CNTACT.EQ.231.OR.CNTACT.EQ.232) THEN
    GO TO 600
  ELSE IF (CNTACT.EQ.322) THEN
    IF (SIGMA(1,2).LT.SIGMA(2,2)) THEN
      GO TO 710
    ELSE
      GO TO 400
    END IF
  ELSE IF (CNTACT.EQ.321) THEN
    GO TO 710
  ELSE IF (DABS(Q(I)).LT.A(I)) THEN
    GO TO 600

```



```

      ELSE
        GO TO 800
      END IF
C
C ---BRANCH HERE IF SHOCK IS TO LEFT OF CONTACT SURFACE---
C ---DETERMINE PROPER ALGORITHM TO USE FOR CALCULATING EXTENDED REIMAN--
C ---VARIABLE CHANGE ALONG CHARACTERISTICS AT THIS NODE-----
C
      ELSE IF (SIGMA(1,1).LT.SIGMA(2,1)) THEN
C
C ---SHOCK ON LEFT, HEADED RIGHT, NO NODE CROSSED---
C
      IF (SHOCK.EQ.322) THEN
        IF (CNTACT.EQ.322.OR.CNTACT.EQ.321) THEN
          GO TO 600
        ELSE IF (CNTACT.EQ.332) THEN
          IF (SIGMA(1,2).GT.SIGMA(2,2)) THEN
            GO TO 700
          ELSE
            GO TO 300
          END IF
        ELSE IF (CNTACT.EQ.331) THEN
          GO TO 700
        ELSE IF (CNTACT.EQ.222.OR.CNTACT.EQ.221) THEN
          GO TO 600
        ELSE IF (CNTACT.EQ.232) THEN
          GO TO 400
        ELSE IF (SIGMA(1,2).GT.SIGMA(2,2)) THEN
          GO TO 710
        ELSE
          GO TO 400
        END IF
      END IF
C
C ---SHOCK ON LEFT, HEADED LEFT, NO NODE CROSSED---
C
      IF (SHOCK.EQ.332) THEN
        IF (CNTACT.EQ.322) THEN
          GO TO 800
        ELSE IF (CNTACT.EQ.332) THEN
          GO TO 300
        ELSE IF (CNTACT.EQ.321) THEN
          GO TO 800
        ELSE IF (CNTACT.EQ.222.OR.CNTACT.EQ.221) THEN
          GO TO 800
        ELSE IF (CNTACT.EQ.231.OR.CNTACT.EQ.232) THEN
          GO TO 400
        ELSE
          GO TO 900
        END IF
      END IF
C
C ---SHOCK ON LEFT, HEADED RIGHT, JUMPS NODE---
C
      IF (SHOCK.EQ.321) THEN
        IF (CNTACT.EQ.322.OR.CNTACT.EQ.321) THEN
          GO TO 600
        ELSE IF (CNTACT.EQ.332.OR.CNTACT.EQ.331) THEN
          GO TO 710
        ELSE IF (CNTACT.EQ.222.OR.CNTACT.EQ.221) THEN
          GO TO 600
        ELSE IF (CNTACT.EQ.231) THEN
          GO TO 710
        ELSE IF (SIGMA(1,2).GT.SIGMA(2,2)) THEN
          GO TO 710
        ELSE
          GO TO 400
        END IF
      END IF

```

```

C
C ---SHOCK ON LEFT, HEADED LEFT, JUMPS NODE---
C
    IF (SHOCK.EQ.331) THEN
        IF (CNTACT.EQ.322.OR.CNTACT.EQ.321) THEN
            GO TO 800
        ELSE IF (CNTACT.EQ.331.OR.CNTACT.EQ.332) THEN
            GO TO 300
        ELSE IF (CNTACT.EQ.221.OR.CNTACT.EQ.222) THEN
            GO TO 800
        ELSE
            GO TO 400
        END IF
    END IF

C
C ---SHOCK ON RIGHT, HEADED RIGHT, NO NODE CROSSED---
C
    IF (SHOCK.EQ.222) THEN
        IF (CNTACT.EQ.222.OR.CNTACT.EQ.221) THEN
            GO TO 600
        ELSE IF (CNTACT.EQ.231) THEN
            GO TO 710
        ELSE IF (CNTACT.EQ.232) THEN
            IF (SIGMA1,2).GT.SIGMA12,2) THEN
                GO TO 700
            ELSE
                GO TO 200
            END IF
        ELSE
            GO TO 900
        END IF
    END IF

C
C ---SHOCK ON RIGHT, HEADED LEFT, NO NODES CROSSED---
C
    IF (SHOCK.EQ.232) THEN
        IF (CNTACT.EQ.222.OR.CNTACT.EQ.221) THEN
            GO TO 800
        ELSE IF (CNTACT.EQ.232) THEN
            IF (DABS(Q1))<.LT.A11) THEN
                GO TO 200
            ELSE
                GO TO 500
            END IF
        ELSE
            GO TO 900
        END IF
    END IF

C
C ---SHOCK ON RIGHT, HEADED RIGHT, JUMPS NODE---
C
    IF (SHOCK.EQ.221) THEN
        IF (CNTACT.EQ.221.OR.CNTACT.EQ.222) THEN
            GO TO 600
        ELSE IF (CNTACT.EQ.231) THEN
            GO TO 710
        ELSE IF (CNTACT.EQ.232) THEN
            GO TO 700
        ELSE
            GO TO 900
        END IF
    END IF

C
C ---SHOCK ON RIGHT, HEADED LEFT, JUMPS NODE---
C
    IF (CNTACT.EQ.221.OR.CNTACT.EQ.222) THEN
        GO TO 800
    ELSE IF (CNTACT.EQ.231.OR.CNTACT.EQ.232) THEN
        GO TO 400
    
```

```

      ELSE
        GO TO 900
      END IF
C
C ---SHOCK AND CONTACT SURFACE ARE AT THE SAME LOCATION AFTER TIME---
C ---ZERO---
C
      ELSE IF((SHOCK.EQ.222).AND.(CNTACT.EQ.222))THEN
        SHOCK = 321
        CNTACT = 321
        GO TO 400
      ELSE IF((SHOCK.EQ.322).AND.(CNTACT.EQ.322))THEN
        IF(DABS(Q(I)).LT.A(I)) THEN
          GO TO 300
        ELSE
          GO TO 500
        END IF
      ELSE IF((SHOCK.EQ.332).AND.(CNTACT.EQ.332)) THEN
        SHOCK = 231
        CNTACT = 231
        GO TO 400
      ELSE IF((SHOCK.EQ.232).AND.(CNTACT.EQ.232)) THEN
        IF(DABS(Q(I)).LT.A(I)) THEN
          GO TO 200
        ELSE
          GO TO 500
        END IF
      ELSE
        GO TO 720
      END IF
C
C ---CALL CONDITION SUBROUTINE WHICH CONTAINS ALGORITHM THAT IS---
C ---NUMERICALLY BEST SUITED FOR THE SITUATION AT NODE I---
C
      100 CALL COND1(Q(I),Q(I+1),A(I),A(I+1),RR(I),QQ(I),S(I),DELQQL,
        C      DELRRH,DELSH,DELSL,DELQH,DELAH,DELQL,DELAL,H,EE,
        C      DELT,QQINT,RRINT,SINT,QPRIM,APRIM,AINT,Q(I-1),A(I-1))
        GO TO 1000
C
      200 CALL COND2(Q(I),Q(I-1),A(I),A(I-1),RR(I),QQ(I),S(I),DELQQL,
        C      DELRRL,DELSL,DELQL,DELAL,DELT,H,EE,QQINT,RRINT,SINT,
        C      QPRIM,APRIM,AINT)
        GO TO 1000
C
      300 CALL COND3(Q(I),Q(I+1),A(I),A(I+1),RR(I),QQ(I),S(I),DELQQH,
        C      DELRRH,DELSH,DELQH,DELAH,DELT,H,EE,QQINT,RRINT,SINT,
        C      QPRIM,APRIM,AINT)
        GO TO 1000
C
      400 CALL COND4(I,SHOCK,CNTACT,J,LNODE,RNODE,QQSTEP,RRSTEP,SSTEP)
        GO TO 1100
C
      500 CALL COND5(Q(I),Q(I-1),Q(I+1),A(I),A(I-1),A(I+1),RR(I),QQ(I),
        C      S(I),DELQQL,DELQQH,DELRRL,DELRH,DELSL,DELSH,DELQL,
        C      DELQH,DELAL,DELAH,H,EE,DELT,QQINT,RRINT,SINT,AINT,
        C      QPRIM,APRIM,SHOCK,CNTACT)
        GO TO 1000
C
      600 CALL COND6(SHOCK,CNTACT,MALT)
        QQSTEP=0 00
        RRSTEP=0 00
        SSTEP=0 00
        GO TO 1100
C
      700 CALL COND7(SHOCK,CNTACT,DELT,SIGMA,I2,I,H,MALT,Q(I-1)
        C      QQSTEP=0 00
        C      RRSTEP=0 00
        C      SSTEP=0 00
        C      GO TO 1100

```

```

C
710 CALL COND7N(SHOCK,CNTACT,DELT,SIGMA,I2,I,H,HALT,Q(I),X)
    QQSTEP=0.00
    RRSTEP=0.00
    SSTEP=0.00
    GO TO 1100
C
720 CALL COND7S(SIGMA,HALT,SHOCK,CNTACT)
    QQSTEP=0.00
    RRSTEP=0.00
    SSTEP=0.00
    GO TO 1100
C
800 CALL COND8(SHOCK,CNTACT,HALT)
    QQSTEP=0.00
    RRSTEP=0.00
    SSTEP=0.00
    GO TO 1100
C
900 PRINT *, 'AN IMPOSSIBLE SITUATION EXISTS - ERROR'
    QQSTEP=0.00
    RRSTEP=0.00
    SSTEP=0.00
    HALT = 1
    GOTO 1100
C
C ----- CALCULATE DLTA QQ, DLTA RR & DLTA S
C
1000 DLTAQQ=QQINT-QQ(I)
    DLTARR=RRINT-RR(I)
    DLTAS=SINT-S(I)
C
C ----- CALCULATE Z(K)'S -----
C
    AAVG(1)=(AINT(1)+A(I))/2.0000
    AAVG(3)=(AINT(3)+A(I))/2.0000
    AAVG(2)=0.0000
    SAVG = (SINT+S(I))/2.000
    Z(1)=-((1.0000/G2)*AAVG(1))*((SAVG-G2)*(QPRIM(1)-G2*APRIM(1)))
    Z(3)=(1.0000/G2)*AAVG(3)*((SAVG-G2)*(QPRIM(3)+G2*APRIM(3)))
    Z(2)=0.0000
C
C ----- INTEGRATE THE Z(K)'S -----
C
    INTEG(2)=0.0000
    INTEG(1)=Z(1)*DELT
    INTEG(3)=Z(3)*DELT
C
C ----- SOLVE THE EQUATION -----
C
    QQSTEP=DLTAQQ+INTEG(1)
    RRSTEP=DLTARR+INTEG(3)
    SSTEP=DLTAS+INTEG(2)
C
C ----- STORE THE SOLUTION -----
C
1100 NEHQQ(I)=QQ(I)+QQSTEP
    NEHRR(I)=RR(I)+RRSTEP
    NEHS(I)=S(I)+SSTEP
C
C ----- GO TO NEXT NODE -----
C
    I=I+1
    GOTO 11
1200 CONTINUE
    BORY = 2
    IF(I2(1).EQ.N) THEN
        SK = 1

```

```

ELSE IF(SIGMA(1,2).EQ.3.D00) THEN
  SK = 3
ELSE
  SK = 0
END IF
CALL BONDY(Q(N),Q(N-1),QB0,A(N),A(N-1),QQ(N),QQ(N-1),RR(N),
C   RR(N-1),S(N),S(N-1),H,EE,DELT,RBNDY,RBOPRS,RBOPR,RBODR,
C   RBOTR,J,NEWQQ(N),NEWRR(N),NEWS(N),G,G1,G2,HALT,BDRY,SK)
C
C ----- UPDATE THE VARIABLES -----
C
  I=1
1210 IF (I.EQ.N+1) GOTO 1220
  RR(I)=NEWRR(I)
  QQ(I)=NEWQQ(I)
  S(I)=NEWS(I)
  I=I+1
  GOTO 1210
1220 CONTINUE
  RETURN
  END
C
C *****
C *
C *          CONDITION SUBROUTINES 1-8
C *
C *****
C
C SUBROUTINE COND1(QI,QIPI,AI,AIP1,RR,QQ,S,DELQQL,DELRRH,DELSH,
C   DELSL,DELQH,DELAH,DELQL,DELAL,H,EE,DELT,QQINT,
C   RRINT,SINT,QPRIM,APRIM,AINT,QIM1,AIM1)
C
C *****
C *
C *          SUBROUTINE CONDITION 1
C *
C *****
C
C ----- USED WHEN NO CONTACT SURFACES NOR SHOCKS WITHIN H OF NODE-----
C ----- AND FLOW IS SUBSONIC-----
C ----- CALCULATES QQINT,RRINT,SINT,QPRIM,APRIM-----
C ----- USES FORWARD AND BACKWARD DIFFERENCE SCHEMES-----
C
C ----- VARIABLE DEFINITIONS -----
C
C AI - A(I)
C AIM1 - A(I-1)
C AIP1 - A(I+1)
C E(K) - ACTUAL ERROR IN CHARACTERISTIC SLOPE CALCULATION.
C LMD - SLOPE OF THE CHARACTERISTICS (Q+A,Q-A)
C QI - Q(I)
C QIM1 - Q(I-1)
C QIPI - Q(I+1)
C
C DIMENSION LMD(3),DELX(3),QINT(3),AINT(3),E(3),QPRIM(3),APRIM(3)
C INTEGER K
C DOUBLE PRECISION QI,QIPI,AI,AIP1,RR,QQ,S,AIM1,QIM1,
C   DELQQL,DELRRH,DELSH,DELSL,DELQH,DELAH,DELQL,
C   DELAL,H,EE,DELT,LMD,DELX,QINT,AINT,E,
C   QQINT,RRINT,SINT,QPRIM,APRIM
C
C -----INITIAL ESTIMATE OF CHARACTERISTIC SLOPES-----
C
  LMD(1) = QIM1 + AIM1
  LMD(2) = QI
  LMD(3) = QIPI - AIP1
C
C -----CALCULATE LINEARLY INTERPOLATED VALUES OF Q AND A-----

```

```

C
10 K = 1
20 IF (K.LT.4) THEN
    DELX(K) = DELT * LMD(K)
    IF (LMD(K).LT.0.0000) THEN
        QINT(K) = QI - (DELX(K) * DELQH / H)
        AINT(K) = AI - (DELX(K) * DELAH / H)
    ELSE
        QINT(K) = QI - (DELX(K) * DELQL / H)
        AINT(K) = AI - (DELX(K) * DELAL / H)
    END IF
    K = K + 1
    GO TO 20
END IF

C
C -----CALCULATE ERROR BETWEEN ESTIMATED SLOPE AND NEW SLOPE-----
C -----FROM NEW INTERPOLATED VALUES-----
C
E(1) = DABS(LMD(1) - (QINT(1) + AINT(1)))
E(2) = DABS(LMD(2) - (QINT(2)))
E(3) = DABS(LMD(3) - (QINT(3) - AINT(3)))

C
LMD(1) = QINT(1) + AINT(1)
LMD(2) = QINT(2)
LMD(3) = QINT(3) - AINT(3)

C
C -----COMPARE ERROR TO ERROR TOLERANCE LEVEL, ITERATE TIL MET-----
C
IF ((E(1).GT.EE).OR.(E(2).GT.EE).OR.(E(3).GT.EE)) GO TO 10

C
C -----CALCULATE LINEARLY INTERPOLATED VALUES OF REIMAN-----
C -----VARIABLES AND MODIFIED ENTROPY AT POINT A-----
C
QQINT = QQ - (DELX(1) * (DELQQL / H))
IF (LMD(2).LE.0.0000) THEN
    SINT = S - (DELX(2) * (DELSH / H))
ELSE
    SINT = S - (DELX(2) * (DELSL / H))
END IF
RRINT = RR - (DELX(3) * (DELRRH / H))

C
C --- CALCULATE SPATIAL DERIVATIVES ---
C
QPRIM(1) = (QI-QIM1)/H
QPRIM(2) = 0.000
QPRIM(3) = (QIP1-QI)/H
APRIM(1) = (AI-AIM1)/H
APRIM(2) = 0.000
APRIM(3) = (AIP1-AI)/H
RETURN
END

C
SUBROUTINE COND2(QI,QIM1,AI,AIM1,RR,QQ,S,DELQQL,DELRR,DELSL,
C          DELQL,DELAL,DELT,H,EE,QQINT,RRINT,SINT,QPRIM,
C          APRIM,AINT)
C
C *****
C          SUBROUTINE CONDITION 2
C *****

C
C --- BACKWARD DIFFERENCE ALGORITHM FOR CALCULATING ---
C --- RRINT,QQINT,SINT,APRIM,QPRIM,AINT ---
C
DIMENSION LMD(3),DELX(3),QINT(3),AINT(3),E(3),QPRIM(3),APRIM(3)
INTEGER K
DOUBLE PRECISION QI,QIM1,AI,AIM1,RR,QQ,S,DELQQL,DELRR,DELSL,
C          DELQL,DELAL,DELT,H,EE,AINT,QINT,LMD,DELX,E,

```

```

C          QQINT,RRINT,SINT,QPRIM,APRIM
C
C  -----INITIAL ESTIMATE OF CHARACTERISTIC SLOPES-----
C
C      LMD(1) = QIM1 + AIM1
C      LMD(2) = QI
C      LMD(3) = QI - AI
C
C  -----CALCULATE LINEARLY INTERPOLATED VALUES OF Q AND A-----
C
C      10 K = 1
C      20 IF (K.LT.4) THEN
C          DELX(K) = DELT * LMD(K)
C          QINT(K) = QI - (DELX(K) * DELQL / H)
C          AINT(K) = AI - (DELX(K) * DELAL / H)
C          K = K + 1
C          GO TO 20
C      END IF
C
C  -----CALCULATE ERROR BETWEEN ESTIMATED SLOPE AND NEW SLOPE-----
C  -----FROM NEW INTERPOLATED VALUES-----
C
C      E(1) = DABS(LMD(1) - (QINT(1) + AINT(1)))
C      E(2) = DABS(LMD(2) - QINT(2))
C      E(3) = DABS(LMD(3) - (QINT(3) - AINT(3)))
C
C      LMD(1) = QINT(1) + AINT(1)
C      LMD(2) = QINT(2)
C      LMD(3) = QINT(3) - AINT(3)
C
C  -----COMPARE ERROR TO ERROR TOLERANCE LEVEL, ITERATE TIL MET-----
C
C      IF ((E(1).GT.EE).OR.(E(2).GT.EE).OR.(E(3).GT.EE)) GO TO 10
C
C  -----CALCULATE LINEARLY INTERPOLATED VALUES OF REIMAN-----
C  -----VARIABLES AND MODIFIED ENTROPY AT POINT A-----
C
C      QQINT = QQ - (DELX(1) * (DELQQL / H))
C      SINT = S - (DELX(2) * (DELSL / H))
C      RRINT = RR - (DELX(3) * (DELRRL / H))
C
C  --- CALCULATE SPATIAL DERIVATIVES ---
C
C      QPRIM(1) = (QI-QIM1)/H
C      QPRIM(2) = 0.000
C      QPRIM(3) = (QI-QIM1)/H
C      APRIM(1) = (AI-AIM1)/H
C      APRIM(2) = 0.000
C      APRIM(3) = (AI-AIM1)/H
C      RETURN
C      END
C
C  SUBROUTINE COND3(QI,QIP1,AI,AIP1,RR,QQ,S,DELQGH,DELRRH,DELSH,
C      DELQH,DELAH,DELT,H,EE,QQINT,RRINT,SINT,QPRIM,
C      APRIM,AINT)
C
C      *****
C      *
C      *          SUBROUTINE CONDITION 3
C      *
C      *****
C
C  --- FORWARD DIFFERENCE ALGORITHM FOR CALCULATING ---
C  --- RRINT,QQINT,SINT,APRIM,QPRIM,AINT ---
C
C      DIMENSION LMD(3),DELX(3),QINT(3),AINT(3),E(3),QPRIM(3),APRIM(3)
C      INTEGER K
C      DOUBLE PRECISION QI,QIP1,AI,AIP1,RR,QQ,S,
C      DELQGH,DELRRH,DELSH,DELQH,DELAH,

```

```

C          DELT,H,EE,AINT,QINT,LMD,DELX,E,
C          QQINT,RRINT,SINT,QPRIM,APRIM
C
C  -----INITIAL ESTIMATE OF CHARACTERISTIC SLOPES-----
C
C          LMD(1) = QI + AI
C          LMD(2) = QI
C          LMD(3) = QIP1 - AIP1
C
C  -----CALCULATE LINEARLY INTERPOLATED VALUES OF Q AND A-----
C
C          10 K = 1
C          20 IF (K.LT.4) THEN
C              DELX(K) = DELT * LMD(K)
C              QINT(K) = QI - (DELX(K) * DELQH / H)
C              AINT(K) = AI - (DELX(K) * DELAH / H)
C              K = K + 1
C              GO TO 20
C          END IF
C
C  -----CALCULATE ERROR BETWEEN ESTIMATED SLOPE AND NEW SLOPE-----
C  -----FROM NEW INTERPOLATED VALUES-----
C
C          E(1) = DABS(LMD(1) - (QINT(1) + AINT(1)))
C          E(2) = DABS(LMD(2) - QINT(2))
C          E(3) = DABS(LMD(3) - (QINT(3) - AINT(3)))
C
C          LMD(1) = QINT(1) + AINT(1)
C          LMD(2) = QINT(2)
C          LMD(3) = QINT(3) - AINT(3)
C
C  -----COMPARE ERROR TO ERROR TOLERANCE LEVEL, ITERATE TIL MET-----
C
C          IF((E(1).GT.EE).OR.(E(2).GT.EE).OR.(E(3).GT.EE)) GO TO 10
C
C  -----CALCULATE LINEARLY INTERPOLATED VALUES OF REIMAN-----
C  -----VARIABLES AND MODIFIED ENTROPY AT POINT A-----
C
C          QQINT = QQ - (DELX(1) * DELQHM / H)
C          SINT = S - (DELX(2) * DELSH / H)
C          RRINT = RR - (DELX(3) * DELRRH / H)
C
C  --- CALCULATE SPATIAL DERIVATIVES ---
C
C          QPRIM(1) = (QIP1-QI)/H
C          QPRIM(2) = 0.000
C          QPRIM(3) = (QIP1-QI)/H
C          APRIM(1) = (AIP1-AI)/H
C          APRIM(2) = 0.000
C          APRIM(3) = (AIP1-AI)/H
C          RETURN
C          END
C
C  SUBROUTINE COND4(I,SHOCK,CONTACT,J,LNODE,RNODE,QQSTEP,RRSTEP,
C  SSTEP)
C
C          *****
C          "
C          "          SUBROUTINE CONDITION 4
C          "
C          *****
C
C  --- A SITUATION EXISTS AT NODE I THAT WILL BE CORRECTED IN ---
C  --- SUBROUTINE CORRECT. *NODE ARRAYS ARE GIVEN INFORMATION ---
C  --- ON NODE LOCATION AND SHOCK/CONTACT SURFACE PICTURE ----
C
C          ----- VARIABLE DEFINITIONS -----
C
C          CD4TRK - DENOTES HOW MANY NODES NEED TO BE CORRECTED

```



```

C          THIS TIME STEP
C
C          DIMENSION LNODE(4),RNODE(4)
C          INTEGER LNODE,RNODE,I,SHOCK,CNTACT,J,CD4TRK
C          DOUBLE PRECISION QQSTEP,RRSTEP,SSTEP
C -----ASSIGN FIRST NODE ENCOUNTERED DURING THE NEW TIME STEP TO ----
C -----LNODE -----
C
C          IF(LNODE(4).LT.J) THEN
C              CD4TRK = 1
C          END IF
C          IF(CD4TRK.EQ.1) THEN
C              LNODE(1) = I
C              LNODE(2) = SHOCK
C              LNODE(3) = CNTACT
C              LNODE(4) = J
C
C -----IF A SECOND NODE WITH CONDITION 4 IS ENCOUNTERED IN THE -----
C -----SHEEP, THIS NODE IS ASSIGNED TO RNODE -----
C
C              ELSE
C                  RNODE(1) = I
C                  RNODE(2) = SHOCK
C                  RNODE(3) = CNTACT
C                  RNODE(4) = J
C
C              END IF
C
C -----LNODE AND RNODE WILL BE "JUMPED" OVER DURING SHEEP THUS-----
C -----THEIR **STEP VALUES ARE SET TO 0-----
C
C              QQSTEP = 0.000
C              RRSTEP = 0.000
C              SSTEP = 0.000
C
C          IF(CD4TRK.EQ.1) THEN
C              CD4TRK = CD4TRK + 1
C          ELSE
C              CD4TRK = 1
C          END IF
C          RETURN
C          END
C
C          SUBROUTINE COND5(QI,QIM1,QIP1,AI,AIM1,AIP1,RR,QQ,S,DELQQL,
C              DELQQH,DELRL,DELRRH,DELSL,DELSH,DELQL,DELQH,
C              DELAL,DELAH,H,EE,DELT,QQINT,RRINT,SINT,AINT,
C              QPRIM,APRIM,SHOCK,CNTACT)
C
C          *****
C          *
C          *          SUBROUTINE CONDITION 5
C          *
C          *****
C
C --- FOR SUPERSONIC FLOW WITH A DISCONTINUITY ON ONE SIDE OF THE NODE-
C --- CALCULATES QQINT,RRINT,SINT,APRIM,QPRIM,AINT ---
C
C          DIMENSION LMD(3),DELX(3),QINT(3),AINT(3),E(3),QPRIM(3),APRIM(3)
C          INTEGER K,SHOCK,CNTACT
C          DOUBLE PRECISION QI,AI,RR,QQ,S,QIM1,QIP1,AIM1,AIP1,
C              EE,DELT,LMD,DELX,QINT,AINT,E,DELQL,DELQH,DELAL,
C              QQINT,SINT,RRINT,QPRIM,APRIM,DELAH,H,
C              DELQQL,DELQQH,DELRL,DELRRH,DELSL,DELSH
C
C --- DISCONTINUITY ON LEFT, HEADED RIGHT, NOT CROSSING NODE ---
C
C          IF((SHOCK.EQ.322).OR.(CNTACT.EQ.322)) THEN
C
C          -----INITIAL ESTIMATE OF CHARACTERISTIC SLOPES-----

```

```

      LMD(1) = QI + AI
      LMD(2) = QI
      LMD(3) = QIP1 - AIP1
C
C ----CALCULATE LINEARLY INTERPOLATED VALUES OF Q AND A-----
C
      10 K = 1
      20 IF (K.LT.4) THEN
            DELX(K) = DELT * LMD(K)
            QINT(K) = QI - (DELX(K) * DELQH / H)
            AINT(K) = AI - (DELX(K) * DELAH / H)
            K = K + 1
            GO TO 20
      END IF
C
C ----CALCULATE ERROR BETWEEN ESTIMATED SLOPE AND NEW SLOPE-----
C ----FROM NEW INTERPOLATED VALUES-----
C
      E(1) = DABS(LMD(1) - (QINT(1) + AINT(1)))
      E(2) = DABS(LMD(2) - QINT(2))
      E(3) = DABS(LMD(3) - (QINT(3) - AINT(3)))
C
      LMD(1) = QINT(1) + AINT(1)
      LMD(2) = QINT(2)
      LMD(3) = QINT(3) - AINT(3)
C
C ----COMPARE ERROR TO ERROR TOLERANCE LEVEL, ITERATE TO MEET-----
C
      IF((E(1).GT.EE).OR.(E(2).GT.EE).OR.(E(3).GT.EE)) GO TO 10
C
C ----CALCULATE LINEARLY INTERPOLATED VALUES OF REIMAN-----
C ----VARIABLES AND MODIFIED ENTROPY AT POINT A-----
C
      QQINT = QQ - (DELX(1) * DELQQH / H)
      SINT = S - (DELX(2) * DELSH / H)
      RRINT = RR - (DELX(3) * DELRRH / H)
C
C --- CALCULATE SPATIAL DERIVATIVES ---
C
      QPRIM(1) = (QIP1-QI)/H
      QPRIM(2) = 0.000
      QPRIM(3) = (QIP1-QI)/H
      APRIM(1) = (AIP1-AI)/H
      APRIM(2) = 0.000
      APRIM(3) = (AIP1-AI)/H
      END IF
C
C --- DISCONTINUITY ON RIGHT, HEADED LEFT, NOT CROSSING NODE ---
C
      IF((SHOCK.EQ.232).OR.(CONTACT.EQ.232)) THEN
C
C ----INITIAL ESTIMATE OF CHARACTERISTIC SLOPES-----
C
      LMD(1) = QIM1 + AIM1
      LMD(2) = QI
      LMD(3) = QI - AI
C
C ----CALCULATE LINEARLY INTERPOLATED VALUES OF Q AND A-----
C
      30 K = 1
      40 IF (K.LT.4) THEN
            DELX(K) = DELT * LMD(K)
            QINT(K) = QI - (DELX(K) * DELQL / H)
            AINT(K) = AI - (DELX(K) * DELAL / H)
            K = K + 1
            GO TO 40
      END IF
C
C ----CALCULATE ERROR BETWEEN ESTIMATED SLOPE AND NEW SLOPE-----

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```

C -----FROM NEW INTERPOLATED VALUES-----
C
E(1) = DABS(LMD(1) - (QINT(1) + AINT(1)))
E(2) = DABS(LMD(2) - QINT(2))
E(3) = DABS(LMD(3) - (QINT(3) - AINT(3)))
C
LMD(1) = QINT(1) + AINT(1)
LMD(2) = QINT(2)
LMD(3) = QINT(3) - AINT(3)
C
C -----COMPARE ERROR TO ERROR TOLERANCE LEVEL, ITERATE TO MEET-----
C
IF ((E(1).GT.EE).OR.(E(2).GT.EE).OR.(E(3).GT.EE)) GO TO 30
C
C -----CALCULATE LINEARLY INTERPOLATED VALUES OF REIMAN-----
C -----VARIABLES AND MODIFIED ENTROPY AT POINT A-----
C
QQINT = QQ - (DELX(1) * (DELQQL / H))
SINT = S - (DELX(2) * (DELSL / H))
RRINT = RR - (DELX(3) * (DELRRL / H))
C
C --- CALCULATE SPATIAL DERIVATIVES ---
C
QPRIM(1) = (QI-QIM1)/H
QPRIM(2) = 0.000
QPRIM(3) = (QI-QIM1)/H
APRIM(1) = (AI-AIM1)/H
APRIM(2) = 0.000
APRIM(3) = (AI-AIM1)/H
END IF
RETURN
END
C
SUBROUTINE COND6(SHOCK,CNTACT,HALT)
C
C *****
C *
C * SUBROUTINE CONDITION 6
C *
C *****
C
INTEGER SHOCK,CNTACT,HALT
C
PRINT * , 'THE SITUATION FOR CONDITION 6 WITH THE CONTACT SURFACE '
PRINT * , 'TO THE RIGHT OF A SHOCK BOTH HEADED RIGHT OR THE '
PRINT * , 'CONTACT SURFACE TO THE LEFT OF A SHOCK BOTH HEADED LEFT'
PRINT * , 'ADDITIONAL LOGIC IS REQUIRED TO PROCEED'
PRINT * , 'SHOCK =',SHOCK,' CONTACT SURFACE =',CNTACT
HALT = 1
RETURN
END
C
SUBROUTINE COND7(SHOCK,CNTACT,DELT,SIGMA,I2,I,H,HALT,Q)
C
C *****
C *
C * SUBROUTINE CONDITION 7
C *
C *****
C
DIMENSION SIGMA(4,2),I2(4)
INTEGER HALT,SHOCK,CNTACT,I2,I
DOUBLE PRECISION DELT,SIGMA,H,Q
C
PRINT * , 'CONDITION 7 REQUIRES THAT THE CONTACT SURFACE AND '
PRINT * , 'SHOCK(MOVING IN OPPOSITE DIRECTIONS) MEET AND CROSS. '
PRINT * , 'WHEN THEY INTERSECT,THE RESULT IS A FUNCTION OF EXIST-'

```

```

PRINT * , 'ING CONDITIONS AROUND THEM. BOTH THE ORIGINAL SHOCK '
PRINT * , 'AND CONTACT SURFACE WILL EXPERIENCE VELOCITY CHANGES .'
PRINT * , 'THIS SUBROUTINE WOULD HAVE TO CALCULATE WHEN AND WHERE'
PRINT * , 'WITHIN THE TIME/SPACE INTERVAL THE SHOCK AND CONTACT '
PRINT * , 'SURFACE INTERSECT. THIS IS BASED ON KNOWN SPEED AND '
PRINT * , 'THEIR RESPECTIVE LOCATIONS. THIS NEW TIME, DELT(NEW) '
PRINT * , 'THEN COULD BE USED TO RERUN "SWEEP", WITH CONDITION 7S'
PRINT * , 'EXISTING AT TIME = T + DELT(NEW). ADDITIONAL LOGIC IS'
PRINT * , 'REQUIRED TO CONTINUE.'
PRINT * , 'SHOCK =',SHOCK,' CONTACT SURFACE =',CNTACT
PRINT * , 'SIGMA(1,1) =',SIGMA(1,1),' SIGMA(1,2) =',SIGMA(1,2)
PRINT * , 'SIGMA(2,1) =',SIGMA(2,1),' SIGMA(2,2) =',SIGMA(2,2)
PRINT * , 'CONTACT SURFACE VELOCITY =',Q
PRINT * , 'NODE I =',I
HALT = 1
RETURN
END

```

```

SUBROUTINE COND7N(SHOCK,CNTACT,DELT,SIGMA,I2,I,H,HALT,Q,X)

```

```

*****
*                                     *
*           SUBROUTINE CONDITION 7N   *
*                                     *
*****

```

```

DIMENSION SIGMA(4,2),I2(4)
INTEGER SHOCK,CNTACT,I2,I
DOUBLE PRECISION DELT,SIGMA,H,Q,X

```

```

PRINT * , 'CONDITION 7N REQUIRES THAT WHEN EITHER A SHOCK OR '
PRINT * , 'CONTACT SURFACE JUMPS A NODE (AS DETERMINED BY '
PRINT * , 'COMPARING SIGMA(I,1) TO SIGMA(I,2)) A CONTACT SURFACE'
PRINT * , 'OR SHOCK WOULD BE MET AND CROSSED DURING THE JUMP. '
PRINT * , 'THE RESULT WHEN THEY INTERSECT IS A FUNCTION OF '
PRINT * , 'EXISTING CONDITIONS AROUND THEM. BOTH THE ORIGINAL'
PRINT * , 'SHOCK AND THE CONTACT SURFACE WILL EXPERIENCE VELOCITY'
PRINT * , 'CHANGES. THIS SUBROUTINE WOULD HAVE TO CALCULATE WHEN'
PRINT * , 'WITHIN THE TIME INTERVAL AND WHERE SPACIALLY THE '
PRINT * , 'INTERSECTION OCCURS. THIS IS BASED ON KNOWN SPEED AND'
PRINT * , 'THE RESPECTIVE LOCATIONS OF THE SHOCK AND CONTACT '
PRINT * , 'SURFACE. THE NEW TIME, DELT(NEW), COULD THEN BE USED '
PRINT * , 'TO RERUN "SWEEP" WITH CONDITION 4 AT THIS NODE, AND '
PRINT * , 'SIGMA(2,2) = SIGMA(1,2) SO THAT CONDITION 7S WOULD '
PRINT * , 'RESULT IN THE NEXT TIME INTERVAL.'
PRINT * , 'SHOCK =',SHOCK,' CONTACT SURFACE =',CNTACT
PRINT * , 'SIGMA(1,1) =',SIGMA(1,1),' SIGMA(1,2) =',SIGMA(1,2)
PRINT * , 'SIGMA(2,1) =',SIGMA(2,1),' SIGMA(2,2) =',SIGMA(2,2)
PRINT * , 'CONTACT SURFACE VELOCITY =',Q
PRINT * , 'NODE I =',I,' LOCATION AT X =',X
HALT = 1
RETURN
END

```

```

SUBROUTINE COND7S(SIGMA,HALT,SHOCK,CNTACT)

```

```

*****
*                                     *
*           SUBROUTINE CONDITION 7S   *
*                                     *
*****

```

```

DIMENSION SIGMA(4,2)
INTEGER SHOCK,CNTACT,HALT
DOUBLE PRECISION SIGMA

```

```

PRINT * , 'CONDITION 7S HAS BEEN MET. THIS MEANS THAT THE SHOCK '
PRINT * , 'AND CONTACT SURFACE ARE LOCATED AT THE SAME X AT A

```

```

PRINT * , 'TIME OTHER THAN ZERO. THIS SUBROUTINE WOULD HAVE TO '
PRINT * , 'DETERMINE THE RESULT OF THE INTERSECTION BASED ON THE '
PRINT * , 'CONDITIONS TO THE LEFT AND RIGHT. ADDITIONAL LOGIC IS '
PRINT * , 'REQUIRED TO PROCEED.'
PRINT * , 'SHOCK =',SHOCK,' CONTACT SURFACE =',CNTACT
PRINT * , 'SIGMA(1,1) =',SIGMA(1,1),' SIGMA(2,1) =',SIGMA(2,1)
HALT = 1
RETURN
END

C
SUBROUTINE CONDS(SHOCK,CNTACT,HALT)
C
C *****
C *
C * SUBROUTINE CONDITION 8
C *
C *****
C
INTEGER HALT,SHOCK,CNTACT
C
PRINT * , 'CONDITION 8 COULD RESULT ONLY AFTER THE ORIGINAL SHOCK '
PRINT * , 'HAS CROSSED THE CONTACT SURFACE, AND THE SUBSEQUENT '
PRINT * , 'CONDITIONS DETERMINED. ADDITIONAL LOGIC IS REQUIRED TO '
PRINT * , 'CONTINUE.'
PRINT * , 'SHOCK =',SHOCK,' CONTACT SURFACE =',CNTACT
HALT = 1
RETURN
END

C
SUBROUTINE CORRECT(LNODE,RNODE,N,SIGMA,H,QQ,RR,G,G,G1,G2,I2,X2,M,
C AR,DQ,VS,A,Q)
C
C *****
C *
C * DISCONTINUITY CORRECTION SUBROUTINE
C *
C *****
C
----- VARIABLE DEFINITIONS -----
C
ENTRPy - MODIFIED ENTROPY
C SHOSPD - SONIC VELOCITY
C JA - VELOCITY RELATIVE TO THE SHOCK, LEFT SIDE
C JB - VELOCITY RELATIVE TO THE SHOCK, RIGHT SIDE
C VLCTY - VELOCITY
C
INTEGER LNODE,RNODE,N,I2,MODE,SHOCK,CNTACT,K
DIMENSION LNODE(4),RNODE(4),SIGMA(6,2),QQ(1),RR(1),SINI,I2(4),
C X2(4),AINI,QINI,XA(1),XB(1)
DOUBLE PRECISION SIGMA,QQ,RR,G,A,Q,
C H,G,G1,G2,X2,M,AR,DQ,VS,
C RRA,RRB,QQA,QQB,AA,AB,QA,QB,SA,SB,
C SA1,SA2,VLCTY,SHOSPD,ENTRPy,QQCALC,RRCALC,
C XB,XA
C
C -----DEFINE STATEMENT FUNCTIONS-----
C
QQCALC=VLCTY*SHOSPD*ENTRPy + VLCTY - SHOSPD*ENTRPy
RRCALC=VLCTY*SHOSPD*ENTRPy + VLCTY - SHOSPD*ENTRPy
C
C -----SET INITIAL VALUES OF VARIABLES TO ZERO-----
C
RRA = 0.000
RRB = 0.000
QQA = 0.000
QQB = 0.000
AA = 0.000
AB = 0.000
QA = 0.000

```

```

      QB = 0.000
      SA = 0.000
      SB = 0.000
      DO 15 K=1,4
          XA(K)=0.00
          XB(K)=0.00
15  CONTINUE
C
C -----DETERMINE IF ONLY ONE NODE NEEDS TO BE CORRECTED OR TWO-----
C -----NODES. ALSO, IF SHOCK AND CONTACT SURFACES ARE CLOSE-----
C -----ENOUGH(WITH IN 2H) TO INTERACT-----
C
      IF((LNODE(2).EQ.100).OR.(LNODE(3).EQ.100)) THEN
          IF((LNODE(1).GE.(RNODE(1)-2)).AND.(RNODE(1).GT.0)) THEN
              GO TO 20
          ELSE
              GO TO 30
          END IF
      ELSE IF(RNODE(1).GT.0) THEN
          GO TO 20
      ELSE
          GO TO 10
      END IF
C
C -----BRANCH HERE IF ONLY ONE NODE TO BE CORRECTED (LNODE) AND,-----
C -----SHOCK/CONTACT SURFACE INTERACTION-----
C
      10 SHOCK = LNODE(2)
         CNTACT = LNODE(3)
C
C ---SHOCK ON LEFT, HEADED RIGHT, JUMPS OR NOT, CONTACT SURFACE ON-----
C ---RIGHT, HEADED LEFT, DOESN'T CROSS NODE ---
C
      IF((SHOCK.EQ.322.OR.SHOCK.EQ.321).AND.(CNTACT.EQ.232)) THEN
C
C -----DETERMINE NODE TO RIGHT OF SHOCK AND CONTACT SURFACE-----
C
         I2(1) = LNODE(1) + 1
         I2(2) = LNODE(1) + 1
         CALL DELTAX(I2,SIGMA,XB,XA,H)
C
C -----EXTRAPOLATE TO RIGHT FACE OF CONTACT SURFACE-----
C
         IF (I2(2).EQ.N) THEN
             CALL BBDRY(RR(N),QQ(N),S(N),RRB,QQB,SB,AB,QB)
         ELSE
             CALL EXTRAP(RR(I2(2)),RR(I2(2)+1),QQ(I2(2)),QQ(I2(2)+1),
C
C               S(I2(2)),S(I2(2)+1),XB(2),H,RRB,QQB,SB,AB,QB)
             END IF
C
C -----CALCULATE VARIABLE CHANGE ACROSS CONTACT SURFACE-----
C
         SA = S(I2(2)-1)
         CALL CSJUMP(AB,QB,SB,SA,G2,QA,AA)
C
C -----IF SHOCK INTERACTS CALCULATE CHANGE IN VARIABLES ACROSS SHOCK-----
C
         IF(SHOCK.EQ.321) THEN
             QB = QA
             AB = AA
             SB = SA
             CALL CSJUMP(AB,QB,SB,AR,DQ,VS,G,G1,A,AA,QA,SA)
             QQA = QGCALC(QA,AA,SA)
             RRA = RRCALC(QA,AA,SA)
C
C -----INTERPOLATE TO NODE(I2(1)-1) AND ASSIGN CORRECTED VALUES-----
C
             IF (I2(1).EQ.2) THEN
                 CALL BBDRY(RRA,QQA,SA,RR(I2(1)-1),QQ(I2(1)-1),

```

```

C          S(I2(1)-1),A(I2(1)-1),Q(I2(1)-1))
C      ELSE
C      CALL INTERP(RRA,RR(I2(1)-2),QQA,QQ(I2(1)-2),SA,
C          S(I2(1)-2),XA(1),(XA(1)+H),RR(I2(1)-1),
C          QQ(I2(1)-1),S(I2(1)-1),A(I2(1)-1),
C          Q(I2(1)-1))
C      END IF
C      ELSE
C          QQ(I2(2)-1) = QQCALC(QA,AA,SA)
C          RR(I2(2)-1) = RRCALC(QA,AA,SA)
C          S(I2(2)-1) = SA
C          A(I2(2)-1) = AA
C          Q(I2(2)-1) = QA
C      END IF
C
C ----SHOCK ON RIGHT, HEADED LEFT, CROSSES NODE OR NOT; CONTACT SURFACE-
C ----ON LEFT,HEADED RIGHT, DOES'NT CROSS NODE---
C
C      ELSE IF((SHOCK.EQ.232.OR.SHOCK.EQ.231).AND.(CNTACT.EQ.322)) THEN
C
C -----DETERMINE NODE TO RIGHT TO SHOCK AND CONTACT SURFACE-----
C
C          I2(1) = LNODE(1)
C          I2(2) = LNODE(1)
C          CALL DELTAX(I2,SIGMA,XB,XA,H)
C
C -----EXTRAPOLATE TO LEFT FACE OF CONTACT SURFACE-----
C
C          IF (I2(2).EQ.2) THEN
C              CALL BBDY(RR(1),QQ(1),S(1),RRA,QQA,SA,AA,QA)
C          ELSE
C              CALL EXTRAP(RR(I2(2)-1),RR(I2(2)-2),QQ(I2(2)-1),QQ(I2(2)-2),
C                  S(I2(2)-1),S(I2(2)-2),XA(2),H,RRA,QQA,SA,AA,QA)
C          END IF
C
C -----CALCULATE VARIABLE CHANGE ACROSS CONTACT SURFACE-----
C
C          SA = S(I2(2))
C          CALL CSJUMP(AA,QA,SA,SB,G2,QB,AB)
C
C -----IF SHOCK INTERACTS CALCULATE CHANGE IN VARIABLES ACROSS SHOCK-----
C
C          IF(SHOCK.EQ.231) THEN
C              QA = QB
C              AA = AB
C              SA = SB
C              CALL CKJUMP(AA,QA,SA,AR,DQ,VS,G,G1,M,AB,QB,SB)
C              QQB = QQCALC(QB,AB,SB)
C              RRB = RRCALC(QB,AB,SB)
C
C -----INTERPOLATE TO NODE(I2(1)) AND ASSIGN CORRECTED VALUES-----
C
C          IF (I2(1).EQ.N) THEN
C              CALL BBDY(RRB,QQB,SB,RR(I2(1)),QQ(I2(1)),S(I2(1)),
C                  A(I2(1)),Q(I2(1)))
C          ELSE
C              CALL INTERP(RRB,RR(I2(1)+1),QQB,QQ(I2(1)+1),SB,
C                  S(I2(1)+1),XB(1),(XB(1)+H),RR(I2(1)),
C                  QQ(I2(1)),S(I2(1)),A(I2(1)),Q(I2(1)))
C          END IF
C      ELSE
C          QQ(I2(2)) = QQCALC(QB,AB,SB)
C          RR(I2(2)) = RRCALC(QB,AB,SB)
C          S(I2(2)) = SB
C          A(I2(2)) = AB
C          Q(I2(2)) = QB
C      END IF
C
C      SHOCK ON RIGHT,HEADED RIGHT, DOESN'T CROSS OR ON LEFT,HEADED---
```

```

C --- RIGHT, DOES CROSS, AND CONTACT ON LEFT, HEADED RIGHT, CROSSES---
C --- OR NOT: OR SHOCK ON RIGHT, HEADED LEFT, DOESN'T CROSS NODE ---
C --- WITH CONTACT ON LEFT, HEADED RIGHT, AND CROSSED NODE ---
C
      ELSE IF(((SHOCK.EQ.222.OR.SHOCK.EQ.321).AND.(CONTACT.EQ.321.OR.
C          CONTACT.EQ.322)).OR.(SHOCK.EQ.232.AND.CONTACT.EQ.321))
C          THEN
C
C      -----DETERMINE NODE TO RIGHT OF SHOCK AND CONTACT SURFACE-----
C
C          I2(1) = LNODE(1) + 1
C          I2(2) = LNODE(1) + 1
C          CALL DELTAX(I2,SIGMA,XB,XA,H)
C
C      -----EXTRAPOLATE TO RIGHT FACE OF SHOCK-----
C
C          IF (I2(1).EQ.N) THEN
C              CALL BBDY(RR(N),QQ(N),S(N),RRB,QQB,SB,AB,QB)
C          ELSE
C              CALL EXTRAP(RR(I2(1)),RR(I2(1)+1),QQ(I2(1)),QQ(I2(1)+1),
C                  S(I2(1)),S(I2(1)+1),XB(1),H,RRB,QQB,SB,AB,QB)
C          END IF
C
C      -----CALCULATE CHANGE IN VARIABLES ACROSS SHOCK-----
C
C          IF(SHOCK.EQ.232) THEN
C              AA = AB/AR
C              SA1 = (G1/G)*DLOG((2.000*G*(H**2)-G+1.000) / (G+1.000))
C              SA2 = G1*DLOG(((G-1.000)*(H**2)+2.000) / ((G+1.000)*(
C                  H**2)))
C              SA = SB+SA1+SA2
C              UB = QB - VS
C              UA = UB -AA*DQ
C              QA = UA + VS
C          ELSE
C              CALL SKJUMP(AB,QB,SB,AR,DQ,VS,G,G1,H,AA,QA,SA)
C          END IF
C
C      -----CALCULATE CHANGE IN VARIABLES ACROSS CONTACT SURFACE-----
C
C          IF(CONTACT.EQ.322) THEN
C              QQ(I2(1)-1) = QGCALC(QA,AA,SA)
C              RR(I2(1)-1) = RRCALC(QA,AA,SA)
C              S(I2(1)-1) = SA
C              A(I2(1)-1) = AA
C              Q(I2(1)-1) = QA
C          ELSE
C              QB = QA
C              AB = AA
C              SB = SA
C              SA = S(I2(2)-2)
C              CALL CSJUMP(AB,QB,SB,SA,G2,QA,AA)
C              QQA = QGCALC(QA,AA,SA)
C              RRA = RRCALC(QA,AA,SA)
C
C      -----INTERPOLATE TO NODE(I2(2)-1) AND ASSIGN CORRECTED VALUES-----
C
C          IF (I2(2).EQ.2) THEN
C              CALL BBDY(RRA,QQA,SA,RR(I2(2)-1),QQ(I2(2)-1),
C                  S(I2(2)-1),A(I2(2)-1),Q(I2(2)-1))
C          ELSE
C              CALL INTERP(RRA,RR(I2(2)-2),QQA,QQ(I2(2)-2),SA,
C                  S(I2(2)-2),XA(2),(XA(2)+H),RR(I2(2)-1),
C                  QQ(I2(2)-1),S(I2(2)-1),A(I2(2)-1),
C                  Q(I2(2)-1))
C          END IF
C
C      END IF
C
C      ----SHOCK ON LEFT, HEADED LEFT, DOESN'T CROSS NODE, OR ON RIGHT,---

```



```

C ---HEADED LEFT, CROSSED NODE, AND CONTACT ON RIGHT, HEADED LEFT,---
C ---CROSSES NODE OR NOT: OR SHOCK ON LEFT, HEADED RIGHT, DOESN'T---
C ---CROSS NODE, AND CONTACT ON RIGHT, HEADED LEFT, CROSSED NODE---
C
      ELSE IF((SHOCK.EQ.332.OR.SHOCK.EQ.231).AND.(CONTACT.EQ.232.OR.
C          CONTACT.EQ.231)).OR.(SHOCK.EQ.322.AND.CONTACT.EQ.231))
C          THEN
C
C -----DETERMINE NODE TO RIGHT OF SHOCK AND CONTACT SURFACE-----
C
C          I2(1) = LNODE(1)
C          I2(2) = LNODE(1)
C          CALL DELTAX(I2,SIGMA,XB,XA,H)
C
C -----EXTRAPOLATE TO LEFT FACE OF SHOCK-----
C
C          IF (I2(1).EQ.2) THEN
C              CALL BBDY(RR(1),QQ(1),S(1),RRA,QQA,SA,AA,QA)
C          ELSE
C              CALL EXTRAP(RR(I2(1)-1),RR(I2(1)-2),QQ(I2(1)-1),QQ(I2(1)-2),
C                  S(I2(1)-1),S(I2(1)-2),XA(1),H,RRA,QQA,SA,AA,QA)
C          END IF
C
C -----CALCULATE CHANGE IN VARIABLES ACROSS SHOCK-----
C
C          IF(SHOCK.EQ.322) THEN
C              AB = AA/AR
C              SA1 = (G1/G)*LOG((2.000*(H**2)-G+1.000) / (G+1.000))
C              SA2 = G1*LOG((G-1.000)*(H**2)+2.000) / ((G+1.000)*
C                  (H**2))
C              SB = SA+SA1+SA2
C              UA = QA - VS
C              UB = UA - AB*DQ
C              QB = UB + VS
C          ELSE
C              CALL SKJUMP(AA,QA,SA,AR,DQ,VS,G,G1,H,AB,QB,SB)
C          END IF
C
C -----CALCULATE CHANGE IN VARIABLES ACROSS CONTACT SURFACE-----
C
C          IF(CONTACT.EQ.232) THEN
C              QQ(I2(1)) = QQCALC(QB,AB,SB)
C              RR(I2(1)) = RRCALC(QB,AB,SB)
C              S(I2(1)) = SB
C              A(I2(1)) = AB
C              Q(I2(1)) = QB
C          ELSE
C              QA = QB
C              AA = AB
C              SA = SB
C              SB = S(I2(2)+1)
C              CALL CSJUMP(AA,QA,SA,SB,G2,QB,AB)
C              QQB = QQCALC(QB,AB,SB)
C              RRB = RRCALC(QB,AB,SB)
C
C -----INTERPOLATE TO NODE(I2(2)) AND ASSIGN CORRECTED VALUES-----
C
C          IF (I2(2).EQ.N) THEN
C              CALL BBDY(RRB,QQB,SB,RR(I2(2)),QQ(I2(2)),S(I2(2)),
C                  A(I2(2)),Q(I2(2)))
C          ELSE
C              CALL INTERP(RRB,RR(I2(2)+1),QQB,QQ(I2(2)+1),SB,
C                  S(I2(2)+1),XB(2),(XB(2)+H),RR(I2(2)),
C                  QQ(I2(2)),S(I2(2)),A(I2(2)),Q(I2(2)))
C          END IF
C
C          END IF
C          GO TO 40
C

```

```

C -----BRANCH HERE IF LNODE AND RNODE ARE CLOSE ENOUGH FOR SHOCK-----
C -----AND CONTACT SURFACE INTERACTION-----
C
C LEFT NODE:
C ---SHOCK ON RIGHT, HEADED RIGHT, JUMPS NODE WITH CONTACT ON LEFT---
C ---HEADED RIGHT, JUMPS NODE OR NOT, OR NO SHOCK WITH CONTACT ON ---
C ---LEFT, HEADED RIGHT, JUMPS NODE---
C RIGHT NODE:
C ---SHOCK ON LEFT, HEADED RIGHT, JUMPS NODE WITH NO CONTACT SURFACE---
C
  20 IF((LNODE(2).EQ.221).AND.(LNODE(3).EQ.321.OR.LNODE(3).EQ.322))
    C .OR.(LNODE(2).EQ.100.AND.LNODE(3).EQ.321)) THEN
      IF((RNODE(2).EQ.321).AND.(RNODE(3).EQ.100)) THEN
C -----DETERMINE NODE TO RIGHT OF SHOCK AND CONTACT SURFACE-----
C
        I2(1) = RNODE(1) + 1
        IF(LNODE(2).EQ.322) THEN
          I2(2) = LNODE(1)
        ELSE
          I2(2) = LNODE(1) + 1
        END IF
        CALL DELTAX(I2,SIGMA,XB,XA,H)
C
C -----EXTRAPOLATE TO RIGHT FACE OF SHOCK-----
C
        IF (I2(1).EQ.N) THEN
          CALL BBDRY(RRIN),QQ(N),S(N),RRB,QQB,SB,AB,QB)
        ELSE
          CALL EXTRAP(RR(I2(1)),RR(I2(1)+1),QQ(I2(1)),QQ(I2(1)+1),
C          S(I2(1)),S(I2(1)+1),XB(1),H,RRB,QQB,SB,AB,QB)
          END IF
C
C -----CALCULATE VARIABLE CHANGE ACROSS SHOCK-----
C
        CALL SKJUMPI(AB,QB,SB,AR,OQ,VS,G,G1,M,AA,QA,SA)
C
C -----DETERMINE CORRECTED VALUES AT NODE(I2(1)-1) AND ASSIGN THEM-----
C
        IF(LNODE(2).EQ.100) THEN
          QQA = QQCALC(QA,AA,SA)
          RRA = RRCALC(QA,AA,SA)
C
C -----INTERPOLATE TO NODE(I2(1)-1) AND ASSIGN CORRECTED VALUES-----
C
          IF (I2(1).EQ.2) THEN
            CALL BBDRY(RRA,QQA,SA,RR(I2(1)-1),QQ(I2(1)-1),
C            S(I2(1)-1),A(I2(1)-1),Q(I2(1)-1))
          ELSE
            CALL INTERP(RRA,RR(I2(1)-2),QQA,QQ(I2(1)-2),SA,
C            S(I2(1)-2),XA(1),XA(1)+H,RR(I2(1)-1),
C            QQ(I2(1)-1),S(I2(1)-1),A(I2(1)-1),
C            Q(I2(1)-1))
          END IF
C
C -----EXTRAPOLATE TO RIGHT FACE OF CONTACT SURFACE-----
C
          IF (I2(2).EQ.N) THEN
            CALL BBDRY(RRIN),QQ(N),S(N),RRB,QQB,SB,AB,QB)
          ELSE
            CALL EXTRAP(RR(I2(2)),RR(I2(2)+1),QQ(I2(2)),QQ(I2(2)+1),
C            S(I2(2)),S(I2(2)+1),XB(2),H,RRB,QQB,SB,AB,QB)
            END IF
          ELSE
            QQ(I2(1)-1) = QQCALC(QA,AA,SA)
            RR(I2(1)-1) = RRCALC(QA,AA,SA)
            S(I2(1)-1) = SA
            A(I2(1)-1) = AA
            Q(I2(1)-1) = QA

```

```

      IF(LNODE(3).EQ.322) THEN
        QQ(I2(2)) = QQ(I2(1)-1)
        RR(I2(2)) = RR(I2(1)-1)
        S(I2(2)) = S(I2(1)-1)
        A(I2(2)) = A(I2(1)-1)
        Q(I2(2)) = Q(I2(1)-1)
        GO TO 40
      ELSE
        QB = QA
        AB = AA
        SB = SA
      END IF
    END IF
    SA = S(I2(2)-2)
    CALL CSJUMP(AB,QB,SB,SA,G2,QA,AA)
    QQA = QGCALC(QA,AA,SA)
    RRA = RRCALC(QA,AA,SA)
C
C -----INTERPOLATE TO NODE(I2(2)-1) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(2).EQ.2) THEN
        CALL BBDRY(RRA,QQA,SA,RR(I2(2)-1),QQ(I2(2)-1),
          S(I2(2)-1),A(I2(2)-1),Q(I2(2)-1))
      ELSE
        CALL INTERP(RRA,RR(I2(2)-2),QQA,QQ(I2(2)-2),SA,
          S(I2(2)-2),XA(2),(XA(2)+H),RR(I2(2)-1),
          QQ(I2(2)-1),S(I2(2)-1),A(I2(2)-1),
          Q(I2(2)-1))
      END IF
    END IF
C
C LEFT NODE:
C ---SHOCK ON LEFT, HEADED RIGHT, JUMPS NODE WITH NO CONTACT---
C RIGHT NODE:
C ---NO SHOCK, WITH CONTACT ON RIGHT, HEADED LEFT, JUMPS NODE---
C
      ELSE IF((LNODE(2).EQ.321).AND.(LNODE(3).EQ.100)) THEN
        IF((RNODE(2).EQ.100).AND.(RNODE(3).EQ.231)) THEN
C
C -----DETERMINE NODE TO RIGHT OF SHOCK AND CONTACT SURFACE-----
C
          I2(1) = LNODE(1) + 1
          I2(2) = RNODE(1)
          CALL DELTAX(I2,SIGMA,XB,XA,H)
C
C -----CALCULATE JUMP THROUGH CONTACT SURFACE THEN SHOCK-----
C
          IF(I2(1).EQ.(I2(2)-1)) THEN
            AA = A(I2(1))
            QA = Q(I2(1))
            SA = S(I2(1))
            SB = S(I2(2)+1)
            CALL CSJUMP(AA,QA,SA,SB,G2,QB,AB)
            QQB = QGCALC(QB,AB,SB)
            RRB = RRCALC(QB,AB,SB)
C
C -----INTERPOLATE TO NODE(I2(2)) AND ASSIGN CORRECTED VALUES-----
C
            IF (I2(2).EQ.N) THEN
              CALL BBDRY(RRB,QQB,SB,RR(I2(2)),QQ(I2(2)),S(I2(2)),
                A(I2(2)),Q(I2(2)))
            ELSE
              CALL INTERP(RRB,RR(I2(2)+1),QQB,QQ(I2(2)+1),SB,
                S(I2(2)+1),XB(2),(XB(2)+H),RR(I2(2)),
                QQ(I2(2)),S(I2(2)),A(I2(2)),Q(I2(2)))
            END IF
            AB = A(I2(1))
            QB = Q(I2(1))
            SB = S(I2(1))

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```

      CALL SKJUMP(AB,GB,SB,AR,DQ,VS,G,G1,M,AA,QA,SA)
      QQA = QQCALC(QA,AA,SA)
      RRA = RRCALC(QA,AA,SA)
C
C -----INTERPOLATE TO NODE(I2(1)-1) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(1).EQ.2) THEN
      CALL BBORY(RRA,QQA,SA,RR(I2(1)-1),QQ(I2(1)-1),
      C      S(I2(1)-1),A(I2(1)-1),Q(I2(1)-1))
      ELSE
      CALL INTERP(RRA,RR(I2(1)-2),QQA,QQ(I2(1)-2),SA,
      C      S(I2(1)-2),XA(1),XA(1)+H,RR(I2(1)-1),
      C      QQ(I2(1)-1),S(I2(1)-1),A(I2(1)-1),
      C      Q(I2(1)-1))
      END IF
      ELSE
      RR(I2(1)-1) = RR(I2(1)-2)
      QQ(I2(1)-1) = QQ(I2(1)-2)
      S(I2(1)-1) = S(I2(1)-2)
      A(I2(1)-1) = A(I2(1)-2)
      Q(I2(1)-1) = Q(I2(1)-2)
      RR(I2(2)) = RR(I2(2)+1)
      QQ(I2(2)) = QQ(I2(2)+1)
      S(I2(2)) = S(I2(2)+1)
      A(I2(2)) = A(I2(2)+1)
      Q(I2(2)) = Q(I2(2)+1)
      END IF
      END IF
C
C RIGHT NODE:
C ---SHOCK ON RIGHT, HEADED LEFT, JUMPS NODE WITH NO CONTACT---
C LEFT NODE:
C ---NO SHOCK, WITH CONTACT ON LEFT, HEADED RIGHT, JUMPS NODE---
C
      ELSE IF (RNODE(2).EQ.231).AND.(RNODE(3).EQ.100) THEN
      IF (LNODE(2).EQ.100).AND.(LNODE(3).EQ.321) THEN
C
C -----DETERMINE NODE TO RIGHT TO SHOCK AND CONTACT SURFACE-----
C
      I2(1) = RNODE(1)
      I2(2) = LNODE(1) + 1
      CALL DELTAX(I2,SIGMA,XB,XA,H)
C
C -----CALCULATE JUMP THROUGH CONTACT SURFACE THEN SHOCK-----
C
      IF (I2(2).EQ.(I2(1)-1)) THEN
      AB = A(I2(2))
      GB = Q(I2(2))
      SB = S(I2(2))
      SA = S(I2(2)-2)
      CALL CSJUMP(AB,GB,SB,SA,G2,QA,AA)
      QQA = QQCALC(QA,AA,SA)
      RRA = RRCALC(QA,AA,SA)
C
C -----INTERPOLATE TO NODE(I2(2)-1) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(2).EQ.2) THEN
      CALL BBORY(RRA,QQA,SA,RR(I2(2)-1),QQ(I2(2)-1),
      C      S(I2(2)-1),A(I2(2)-1),Q(I2(2)-1))
      ELSE
      CALL INTERP(RRA,RR(I2(2)-2),QQA,QQ(I2(2)-2),SA,
      C      S(I2(2)-2),XA(2),XA(2)+H,RR(I2(2)-1),
      C      QQ(I2(2)-1),S(I2(2)-1),A(I2(2)-1),
      C      Q(I2(2)-1))
      END IF
      AA = A(I2(2))
      QA = Q(I2(2))
      SA = S(I2(2))
      CALL SKJUMP(AA,QA,SA,AR,DQ,VS,G,G1,M,AB,GB,SB)

```

```

      QOB = QOCALC(QB,AB,SB)
      RRB = RRCALC(QB,AB,SB)
C
C -----INTERPOLATE TO NODE(I2(1)) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(1).EQ.M) THEN
        CALL BBDRY(RRB,QOB,SB,RR(I2(1)),QQ(I2(1)),S(I2(1)),
          A(I2(1)),Q(I2(1)))
      ELSE
        CALL INTERP(RRB,RR(I2(1)+1),QOB,QQ(I2(1)+1),SB,
          S(I2(1)+1),XB(1),(XB(1)+M),RR(I2(1)),
          QQ(I2(1)),S(I2(1)),A(I2(1)),Q(I2(1)))
      END IF
    ELSE
      RR(I2(2)-1) = RR(I2(2)-2)
      QQ(I2(2)-1) = QQ(I2(2)-2)
      S(I2(2)-1) = S(I2(2)-2)
      A(I2(2)-1) = A(I2(2)-2)
      Q(I2(2)-1) = Q(I2(2)-2)
      RR(I2(1)) = RR(I2(1)+1)
      QQ(I2(1)) = QQ(I2(1)+1)
      S(I2(1)) = S(I2(1)+1)
      A(I2(1)) = A(I2(1)+1)
      Q(I2(1)) = Q(I2(1)+1)
    END IF
  END IF
C
C RIGHT NODE:
C ---SHOCK ON LEFT, HEADED LEFT, JUMPS NODE WITH CONTACT ON RIGHT---
C ---HEADED LEFT, DOES NOT CROSS NODE OR NO SHOCK WITH CONTACT ON---
C ---RIGHT, HEADED LEFT, CROSSED NODE---
C LEFT NODE:
C ---SHOCK ON RIGHT, HEADED LEFT, JUMPS NODE WITH NO CONTACT---
C
      ELSE IF (LRNODE(2).EQ.331) AND (LRNODE(3).EQ.231 OR LRNODE(3).EQ.
        232) OR (LRNODE(2).EQ.100 AND LRNODE(3).EQ.231) THEN
        IF (LRNODE(2).EQ.231) AND (LRNODE(3).EQ.100) THEN
C
C -----DETERMINE NODE TO RIGHT OF SHOCK AND CONTACT SURFACE-----
C
          I2(1) = LRNODE(1)
          IF LRNODE(3).EQ.232 THEN
            I2(2) = LRNODE(1) + 1
          ELSE
            I2(2) = LRNODE(1)
          END IF
          CALL DELTAX(I2,SIGMA,XB,XA,M)
C
C -----EXTRAPOLATE TO LEFT FACE OF SHOCK-----
C
          IF (I2(1).EQ.2) THEN
            CALL BBDRY(RR(1),QQ(1),S(1),RRA,QQA,SA,AA,QA)
          ELSE
            CALL EXTRAP(RR(I2(1)-1),RR(I2(1)-2),QQ(I2(1)-1),QQ(I2(1)-2),
              S(I2(1)-1),S(I2(1)-2),XA(1),M,RRA,QQA,SA,AA,QA)
          END IF
C
C -----CALCULATE VARIABLE CHANGE ACROSS SHOCK-----
C
          CALL SKJUMP(AA,QA,SA,AR,DQ,VS,G,G1,M,AB,QB,SB)
C
C -----DETERMINE CORRECTED VALUES AT NODE(I2(1)) AND ASSIGN THEM-----
C
          IF LRNODE(2).EQ.100 THEN
            QOB = QOCALC(QB,AB,SB)
            RRB = RRCALC(QB,AB,SB)
C
C -----INTERPOLATE TO NODE(I2(1)) AND ASSIGN CORRECTED VALUES-----

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      IF (I2(1).EQ.N) THEN
      CALL BBORY(RRB,QQB,SB,RR(I2(1)),QQ(I2(1)),S(I2(1)),
C      A(I2(1)),Q(I2(1)))
      ELSE
      CALL INTERP(RRB,RR(I2(1)+1),QQB,QQ(I2(1)+1),SB,
C      S(I2(1)+1),XB(1),(XB(1)+H),RR(I2(1)),
C      QQ(I2(1)),S(I2(1)),A(I2(1)),Q(I2(1)))
      END IF
C
C -----EXTRAPOLATE TO LEFT FACE OF CONTACT SURFACE-----
C
      IF (I2(2).EQ.2) THEN
      CALL BBORY(RR(1),QQ(1),S(1),RRA,QQA,SA,AA,QA)
      ELSE
      CALL EXTRAP(RR(I2(2)-1),RR(I2(2)-2),QQ(I2(2)-1),QQ(I2(2)-2),
C      S(I2(2)-1),S(I2(2)-2),XA(2),H,RRA,QQA,SA,AA,QA)
      END IF
      ELSE
      QQ(I2(1)) = QCALC(QB,AB,SB)
      RR(I2(1)) = RRCALC(QB,AB,SB)
      S(I2(1)) = SB
      A(I2(1)) = AB
      Q(I2(1)) = QB
      IF (RNODE(3).EQ.232) THEN
      RR(I2(1)+1) = RR(I2(1))
      QQ(I2(1)+1) = QQ(I2(1))
      S(I2(1)+1) = S(I2(1))
      A(I2(1)+1) = A(I2(1))
      Q(I2(1)+1) = Q(I2(1))
      GO TO 40
      ELSE
      QA = QB
      AA = AB
      SA = SB
      END IF
      END IF
      SB = S(I2(2)+1)
C
C -----CALCULATE VARIABLE CHANGE ACROSS CONTACT SURFACE-----
C
      CALL CSJUMP(AA,QA,SA,SB,GZ,QB,AB)
      QQB = QCALC(QB,AB,SB)
      RRB = RRCALC(QB,AB,SB)
C
C -----INTERPOLATE TO NODE(I2(2)) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(2).EQ.N) THEN
      CALL BBORY(RRB,QQB,SB,RR(I2(2)),QQ(I2(2)),S(I2(2)),
C      A(I2(2)),Q(I2(2)))
      ELSE
      CALL INTERP(RRB,RR(I2(2)+1),QQB,QQ(I2(2)+1),SB,
C      S(I2(2)+1),XB(2),(XB(2)+H),RR(I2(2)),
C      QQ(I2(2)),S(I2(2)),A(I2(2)),Q(I2(2)))
      END IF
      END IF
      END IF
      GO TO 40
C
C --- BRANCH HERE IF THERE ARE ONE OR TWO NODES TO CORRECT, BUT THEY ---
C --- ARE SEPERATED BY MORE THAN 2H. CHECK FOR SHOCK/CONTACT SURFACE ---
C --- INTERACTION, AND CORRECT APPROPRIATELY ---
C
      30 NODE = LNODE(1)
      SHOCK = LNODE(2)
      CNTACT = LNODE(3)
C
C --- NO SHOCK, CONTACT SURFACE ON LEFT, HEADED RIGHT, JUMPS NODE ---
C
      35 IF((SHOCK.EQ.100).AND.(CNTACT.EQ.321)) THEN

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      IZ(2) = NODE + 1
      CALL DELTAX(IZ,SIGMA,XB,XA,H)
C
C -----CHECK FOR A SHOCK INTERACTING WITH CONTACT SURFACE AT NODE-----
C
      IF((SIGMA(1,2).LT.(X2(2)+H)).AND.
C      (SIGMA(1,2).GT.(X2(2))))THEN
          AB = A(I2(2))
          QB = Q(I2(2))
          SB = S(I2(2))
      ELSE
C
C -----EXTRAPOLATE TO RIGHT FACE OF CONTACT SURFACE-----
C
          IF (I2(2).EQ.N) THEN
              CALL BBDY(RRIN),QQ(N),S(N),RRB,QQB,SB,AB,QB)
          ELSE
              CALL EXTRAP(RR(I2(2)),RR(I2(2)+1),QQ(I2(2)),QQ(I2(2)+1),
C              S(I2(2)),S(I2(2)+1),XB(2),H,RRB,QQB,SB,AB,QB)
          END IF
          END IF
          SA = S(I2(2)-2)
C
C --- CALCULATE VARIABLE CHANGE OVER CONTACT SURFACE ---
C
          CALL CSJUMP(AB,QB,SB,SA,G2,QA,AA)
          QQA = QCALC(QA,AA,SA)
          RRA = RRCALC(QA,AA,SA)
C
C -----INTERPOLATE TO NODE(I2(2)-1) AND ASSIGN CORRECTED VALUES-----
C
          IF (I2(2).EQ.2) THEN
              CALL BBDY(RRA,QQA,SA,RR(I2(2)-1),QQ(I2(2)-1),
C              S(I2(2)-1),A(I2(2)-1),Q(I2(2)-1))
          ELSE
              CALL INTERP(RRA,RR(I2(2)-2),QQA,QQ(I2(2)-2),SA,
C              S(I2(2)-2),XA(2),(XA(2)+H),RR(I2(2)-1),
C              QQ(I2(2)-1),S(I2(2)-1),A(I2(2)-1),
C              Q(I2(2)-1))
          END IF
C
C --- NO SHOCK, CONTACT SURFACE ON LEFT, HEADED LEFT, JUMPS NODE ---
C
          ELSE IF((SHOCK.EQ.100).AND.(CONTACT.EQ.231)) THEN
              IZ(2) = NODE
              CALL DELTAX(IZ,SIGMA,XB,XA,H)
C
C -----CHECK FOR A SHOCK INTERACTING WITH CONTACT SURFACE AT NODE-----
C
              IF((SIGMA(1,2).GT.(X2(2)-(2.000*H))).AND.(SIGMA(1,2).LT.
C              (X2(2)-H))) THEN
                  AA = A(I2(2)-1)
                  QA = Q(I2(2)-1)
                  SA = S(I2(2)-1)
              ELSE
C
C -----EXTRAPOLATE TO LEFT FACE OF CONTACT SURFACE-----
C
                  IF (I2(2).EQ.2) THEN
                      CALL BBDY(RR(1),QQ(1),S(1),RRA,QQA,SA,AA,QA)
                  ELSE
                      CALL EXTRAP(RR(I2(2)-1),RR(I2(2)-2),QQ(I2(2)-1),QQ(I2(2)-2),
C                      S(I2(2)-1),S(I2(2)-2),XA(2),H,RRA,QQA,SA,AA,QA)
                  END IF
                  END IF
                  SB = S(I2(2)+1)
                  CALL CSJUMP(AA,QA,SA,SB,G2,QB,AB)
                  QQB = QCALC(QB,AB,SB)
                  RRB = RRCALC(QB,AB,SB)

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C
C -----INTERPOLATE TO NODE(I2(2)) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(2).EQ.N) THEN
        CALL BDRY(RRB,QQB,SB,RR(I2(2)),QQ(I2(2)),S(I2(2)),
C          A(I2(2)),Q(I2(2)))
      ELSE
        CALL INTERP(RRB,RR(I2(2)+1),QQB,QQ(I2(2)+1),SB,
C          S(I2(2)+1),XB(2),(XB(2)+N),RR(I2(2)),
C          QQ(I2(2)),S(I2(2)),A(I2(2)),Q(I2(2)))
      END IF

C
C --- SHOCK ON LEFT, HEADED RIGHT, JUMPS NODE WITH NO CONTACT ---
C
      ELSE IF((SHOCK.EQ.321).AND.(CONTACT.EQ.100)) THEN
        I2(1) = NODE + 1
        CALL DELTAX(I2,SIGMA,XB,XA,H)

C
C -----CHECK FOR A CONTACT SURFACE INTERACTING WITH SHOCK AT NODE-----
C
      IF((SIGMA(2,2).LT.(X2(1)+H)).AND.
C        (SIGMA(2,2).GT.(X2(1)))) THEN
        AB = A(I2(1))
        QB = Q(I2(1))
        SB = S(I2(1))
      ELSE

C
C -----EXTRAPOLATE TO RIGHT FACE OF SHOCK-----
C
        IF (I2(1).EQ.N) THEN
          CALL BDRY(RR(N),QQ(N),S(N),RRB,QQB,SB,AB,QB)
        ELSE
          CALL EXTRAP(RR(I2(1)),RR(I2(1)+1),QQ(I2(1)),QQ(I2(1)+1),
C            S(I2(1)),S(I2(1)+1),XB(1),H,RRB,QQB,SB,AB,QB)
        END IF
      END IF

C
C --- CALCULATE VARIABLE CHANGE ACROSS SHOCK ---
C
      CALL SKJUMP(AB,QB,SB,AR,DQ,VS,G,G1,M,AA,QA,SA)
      QQA = QQCALC(QA,AA,SA)
      RRA = RRCALC(QA,AA,SA)

C
C -----INTERPOLATE TO NODE(I2(1)-1) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(1).EQ.2) THEN
        CALL BDRY(RRA,QQA,SA,RR(I2(1)-1),QQ(I2(1)-1),
C          S(I2(1)-1),A(I2(1)-1),Q(I2(1)-1))
      ELSE
        CALL INTERP(RRA,RR(I2(1)-2),QQA,QQ(I2(1)-2),SA,
C          S(I2(1)-2),XA(1),(XA(1)+H),RR(I2(1)-1),
C          QQ(I2(1)-1),S(I2(1)-1),A(I2(1)-1),
C          Q(I2(1)-1))
      END IF

C
C --- SHOCK ON RIGHT, HEADED LEFT, JUMPS NODE WITH NO CONTACT SURFACE--
C
      ELSE IF((SHOCK.EQ.231).AND.(CONTACT.EQ.100)) THEN
        I2(1) = NODE
        CALL DELTAX(I2,SIGMA,XB,XA,H)
        IF((SIGMA(2,2).GT.(X2(1)-(2.000*H))).AND.(SIGMA(2,2).LT.
C          (X2(1)-H))) THEN
          AA = A(I2(1)-1)
          QA = Q(I2(1)-1)
          SA = S(I2(1)-1)
        ELSE

C
C -----EXTRAPOLATE TO LEFT FACE OF CONTACT SURFACE-----
C

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```

      IF (I2(1).EQ.2) THEN
        CALL BDDRY(RR(1),QQ(1),S(1),RRA,QA,SA,AA,QA)
      ELSE
        CALL EXTRAP(RR(I2(1)-1),RR(I2(1)-2),QQ(I2(1)-1),QQ(I2(1)-2),
C          S(I2(1)-1),S(I2(1)-2),XA(1),H,RRA,QA,SA,AA,QA)
      END IF
    END IF

C
C --- CALCULATE VARIABLE CHANGE OVER SHOCK ---
C
      IF(I2(1).EQ.N-1) THEN
        END IF
        CALL SKJUMP(AA,QA,SA,AR,DQ,VS,G,G1,H,AB,QB,SB)

C
C -----DETERMINE CORRECTED VALUES AT NODE(I2(1)) AND ASSIGN THEM-----
C
      IF(I2(1).EQ.N-1) THEN
        END IF
        QQB = QQCALC(QB,AB,SB)
        RRB = RRCALC(QB,AB,SB)

C
C -----INTERPOLATE TO NODE(I2(1)) AND ASSIGN CORRECTED VALUES-----
C
      IF (I2(1).EQ.N) THEN
        CALL BDDRY(RRB,QQB,SB,RR(I2(1)),QQ(I2(1)),S(I2(1)),
C          A(I2(1)),Q(I2(1)))
      ELSE
        CALL INTERP(RRB,RR(I2(1)+1),QQB,QQ(I2(1)+1),SB,
C          S(I2(1)+1),XB(1),(XB(1)+H),RR(I2(1)),
C          QQ(I2(1)),S(I2(1)),A(I2(1)),Q(I2(1)))
      END IF
    END IF

C
C --- CHECK IF SECOND NODE NEEDS CORRECTING IF SO LOOP BACK AROUND ---
C
      IF(RNODE(1).EQ.0) GO TO 40
      NODE = RNODE(1)
      SHOCK = RNODE(2)
      CNTACT = RNODE(3)
      RNODE(1) = 0
      GO TO 35

40 RETURN
END

C
SUBROUTINE DELTAX(I2,SIGMA,XB,XA,H)
C
C *****
C *
C * LOCATE DISCONTINUITY WITHIN INTERVAL SUBROUTINE *
C *
C *****
C
  DIMENSION I2(4),SIGMA(4,2),XB(4),XA(4),X2(4)
  INTEGER I2
  DOUBLE PRECISION SIGMA,XB,XA,H,X2
  X2(3)=0.000
  X2(4)=0.000

C
C -----CALCULATE DISTANCE NODE TO RIGHT OF DISCONTINUITY IS FROM LEFT. --
C -----BOUNDARY OF TUBE THEN DETERMINE DISTANCE FROM THIS NODE I2(*)----
C -----TO DISCONTINUITY AND DISTANCE FROM NODE TO LEFT OF DISCONTINUITY-
C -----TO THAT DISCONTINUITY-----
C
  X2(1) = DBLE(I2(1)-1) * H
  XB(1) = (X2(1)-SIGMA(1,2))
  XA(1) = (H-XB(1))

C
C -----CALCULATE SAME VALUES FOR CONTACT SURFACE-----
C

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```

      X2(2) = DBLE(I2(2)-1) * H
      XB(2) = (X2(2)-SIGMA(2,2))
      XA(2) = (H-XB(2))
      RETURN
      END

C
      SUBROUTINE SKJUMP(ADN,QDN,SDN,ARATIO,DELTAQ,VSHOCK,G,G1,M,
C
      AUP,QUP,SUP)
C
      *****
C
      *
C
      *              SHOCK JUMP SUBROUTINE
C
      *
C
      *****
C
      ----- VARIABLE DEFINITIONS -----
C
      *DN - VARIABLE DOWNSTREAM OF SHOCK
C
      ARATIO - SPEED OF SOUND RATIO
C
      DELTAQ - VELOCITY JUMP ACROSS SHOCK
C
      *UP - VARIABLE UPSTREAM OF SHOCK
C
      VSHOCK - SHOCK VELOCITY
C
      DOUBLE PRECISION ADN,QDN,SDN,ARATIO,DELTAQ,VSHOCK,G,G1,M,
C
      AUP,QUP,SUP,SA1,SA2,UDN,UUP
C
      -----CALCULATE THE SPEED OF SOUND CHANGE THRU SHOCK-----
C
      AUP = ADN*ARATIO
C
      -----CALCULATE THE ENTROPY CHANGE THRU SHOCK-----
C
      SA1 = (G1/G)*DLOG((2.000*G*(M**2)-G+1.000)/(G+1.000))
      SA2 = G1*DLOG(((G-1.000)*(M**2)+2)/((G+1.000)*(M**2)))
      SUP = SDN - SA1 - SA2
C
      -----CALCULATE THE VELOCITY CHANGE ACROSS SHOCK-----
C
      UDN = QDN - VSHOCK
      UUP = UDN + ADN*DELTAQ
      QUP = UUP + VSHOCK
      RETURN
      END

C
      SUBROUTINE CSJUMP(ADN,QDN,SDN,SUP,G2,QUP,AUP)
C
      *****
C
      *
C
      *              CONTACT SURFACE JUMP SUBROUTINE
C
      *
C
      *****
C
      DOUBLE PRECISION QUP,QDN,AUP,ADN,SUP,SDN,G2
C
      -----CALCULATE THE SPEED OF SOUND CHANGE ACROSS THE CONTACT SURFACE---
C
      -----NOTE: THE VELOCITY IS CONTINUOUS ACROSS A C.S.-----
C
      QUP = QDN
      AUP = ADN * DEXP((SDN-SUP)/G2)
      RETURN
      END

C
      SUBROUTINE EXTRAP(RRL,RRR,QLL,QQR,SL,SR,DX,DH,RRINT,QQINT,SINT,
C
      AINT,QINT)
C
      *****
C
      *
C
      *              EXTRAPOLATION SUBROUTINE
C
      *
C
      *****

```

```

C      DOUBLE PRECISION RRL,RRR,QQL,QQR,SL,SR,DX,DH,RRINT,QQINT,SINT,
C      AINT,QINT
C
C -----REIMAN VARIABLES ARE EXTRAPOLATED, AND THE CORRESPONDING-----
C -----VALUES FOR VELOCITY AND SPEED OF SOUND ARE CALCULATED-----
C
      RRINT = RRL - (DX * (RRR-RRL)/DH)
      QQINT = QQL - (DX * (QQR-QQL)/DH)
      SINT = SL - (DX * (SR-SL)/DH)
      AINT = (QQINT-RRINT)/(2.000*SINT)
      QINT = (QQINT+RRINT)/(2.000)
      RETURN
      END
C
C      SUBROUTINE INTERP(RRL,RRR,QQL,QQR,SL,SR,DX,DH,RRINT,QQINT,SINT,
C      AINT,QINT)
C      *****
C      *
C      *          INTERPOLATION SUBROUTINE
C      *
C      *****
C
C      DOUBLE PRECISION RRL,RRR,QQL,QQR,SL,SR,DX,DH,RRINT,QQINT,SINT,
C      AINT,QINT
C
C -----REIMAN VARIABLES ARE INTERPOLATED, AND THEIR CORRESPONDING-----
C -----VALUES OF VELOCITY AND SPEED OF SOUND ARE CALCULATED-----
C
      RRINT = RRL - (DX * (RRR-RRL)/DH)
      QQINT = QQL - (DX * (QQR-QQL)/DH)
      SINT = SL - (DX * (SR-SL)/DH)
      AINT = (QQINT-RRINT)/(2.000*SINT)
      QINT = (QQINT+RRINT)/2.000
      RETURN
      END
C
C      SUBROUTINE DBURSTIN,H,QQ,RR,S,G,G1,G2,DELT,I2,X2,W,AR,DQ,VS,
C      #LMPRES,SIGMA,A,Q)
C
C      *****
C      *
C      *          DIAPHRAM BURSTING SUBROUTINE
C      *
C      *****
C
      INTEGER N,I,Y,I2,L,SHKDIR,CSDIR,LMPRES
      DIMENSION X2(4),RR(N),QQ(N),S(N),I2(4),SIGMA(4,2),A(N),Q(N)
      DOUBLE PRECISION X2,X,H,AB,SA,SB,AA,QA,QB,QQA,QQB,RAA,RRB,SIGMA,
C      RR,QQ,S,TS,H,DQ,AR,PR,G,G1,G2,VS,DELT,CSRMN,A,
C      Q,MREIMN,DREIMN,EREIMN,W,SA1,SA2,SAP,SBP,XB,XA
C
C      ***** LOCATING THE NODE TO THE RIGHT *****
C
      DO 10 L=1,4
          SIGMA(L,1)=SIGMA(L,2)
          Y=0
          X=H
          I=2
11      IF (.NOT.(Y.EQ.0)) GOTO 10
          IF (SIGMA(L,1).LT.X) THEN
              X2(L)=X
              I2(L)=I
              Y=1
          END IF
          X=X+H
          I=I+1
          GOTO 11
10      CONTINUE

```

```

C
C ***** CORRECT FOR DIAPHRAM BURSTING *****
C
C
C -----AT TIME ZERO DETERMINE CORRECT SHOCK DIRECTION-----
C -----SHKDIR = 3 IS A SHOCK HEADED LEFT, AND SHKDIR = 2 IS SHOCK-----
C -----TRAVELING RIGHT-----
C
      IF(LMPRES.EQ.3) THEN
            SHKDIR = 3
            X2(1) = X2(1) - H
            I2(1) = I2(1) - 1
            X2(2) = X2(2) - H
            I2(2) = I2(2) - 1
            GO TO 20
      ELSE
            SHKDIR = 2
      END IF
C
20 RRA=RR(I2(1)-1)
   RRB=RR(I2(1))
   QQA=QQ(I2(1)-1)
   QQB=QQ(I2(1))
   SA=S(I2(1)-1)
   SB=S(I2(1))
21 AB=(QQB-RRB)/(2.000*SB)
   AA=(QQA-RRA)/(2.000*SA)
   IF(SHKDIR.EQ.3) THEN
         MREIMN = (RRB-RRA)/AA
         DREMNI = DABS(MREIMN)
   ELSE
         MREIMN = (QQA-QQB)/AB
         DREMNI = MREIMN
   END IF
C
C -----ITERATE FOR PROPER VALUE OF W USING THE QUADRATIC FIT OF THE-----
C -----REIMAN VARIABLE CHANGE WITH V CURVE.  NOTE LEFT MOVING SHOCKS-----
C -----ARE USED IN THESE EQUATIONS SINCE RRB-RRA/AA=-(QQA-QQB/AB)-----
C
100 W=(3.0396408001-((DREMNI+2.7574000)/0.286337000))
   W=5.513294000-LSQRT(W)
   DQ=2.000*(W*W-1.000)/(W*(G+1.000))
   AR=DSQRT(2.000*(G-1.000)*(1.000+((G-1.000)*W*W/2.000))*
C      (G*G2*W*W-1.000))/(G+1.000)*W)
   PR=(2.000*G/(G+1.000))*W*W-((G-1.000)/(G+1.000))
   DR=((G-1.000)*W*W+2.000)/((G+1.000)*W*W)
   EREIMN=DQ+(AR-1.000)*G2-(AR*G1/G)*DLOG(PR*(DR*G))
   IF (DABS(EREIMN-DABS(MREIMN)).LT.0.10-5) GO TO 110
   DREMNI = (DABS(MREIMN) - EREIMN) + DREMNI
   GOTO 100
C
C -----DETERMINE S BEHIND SHOCK AND THEN CALCULATE THE RIEMANN-----
C -----VARIABLE CHANGE ACROSS THE CONTACT SURFACE-----
C
110   SA1 = (G1/G)*DLOG((2.000*G*(W**2)-G+1.000)/(G+1.000))
      SA2 = G1*DLOG(((G-1.000)*(W**2)+2)/((G+1.000)*(W**2)))
      IF(SHKDIR.EQ.2) THEN
            SAP = SB - SA1 - SA2
            SBP = SAP
      ELSE
            SBP = SA - SA1 - SA2
            SAP = SBP
      END IF
103   IF(SHKDIR.EQ.2) THEN
         CSRMN = ((DEXP((SBP-SA1)/G2))*SA1 - SBP)*AR
      ELSE
         CSRMN = ((DEXP((SAP-SB)/G2))*SB - SAP)*AR
      END IF
      IF(SHKDIR.EQ.3) THEN

```